Gr 9 Maths: Content Area 2
Patterns, Algebra & Graphs
QUESTIONS

Mainly past ANA exam content

- Patterns
- Algebraic Expressions
- Factorisation
- Algebraic Equations
- Graphs

PATTERNS

(Solutions on page A1)

The most common weakness that learners have when doing patterns is determining the general term.

What is 'the general term (or rule)'?

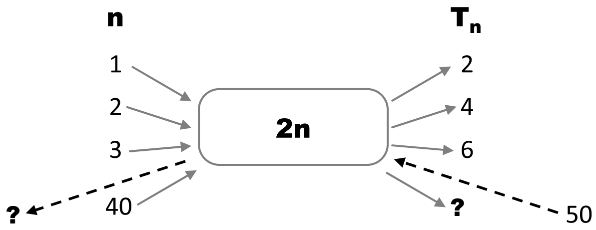
The general term (or rule) of a sequence gives us the value of any term if we know the position.

e.g. If the 'general term' of a sequence is **2n**, we are saying that: the **n**th term is 2**n**

So: the **1**st term is 2(**1**) = 2 ◄
the **2**nd term is 2(**2**) = 4 ◄
the **3**rd term is 2(**3**) = 6 ◄
⋮
& the **40**th term is 2(**40**) = 80 ◄

As we see, any term can be 'generated'.

Note: **n** is the position of the term



In REVERSE: If the term, T_n = 50, what will **n** be?
i.e. Which term has the value 50?
The 25th term! So, **n** = 25.

In TABLE FORM:

n	1	2	3		?		40	
2n					50		?	

50 is the **25**th term ◄
The 40th term is **80** ◄

The Questions

1.1 The next number in the sequence

1 ; 9 ; 25 ; . . . is

A 33 B 36
C 49 D 50 (1)

1.2 The two missing numbers in the sequence below

18 ; 36 ; ____ ; 72 ; ____ ; 108 are

A 38 and 74 B 42 and 78
C 54 and 90 D 45 and 81 (1)

1.3 Which number is missing in the sequence?

$1 \;;\; \frac{1}{2} \;;\; \frac{1}{4} \;;\; . . . \;;\; \frac{1}{16}$

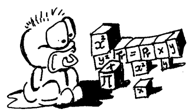
A $\frac{1}{8}$ B $\frac{1}{10}$
C $\frac{1}{12}$ D $\frac{1}{14}$ (1)

1.4 Which number is missing in the number sequence?

$\frac{1}{3} \;;\; . . . \;;\; \frac{1}{12} \;;\; \frac{1}{24} \;;\; \frac{1}{48}$

A $\frac{1}{6}$ B $\frac{1}{8}$
C $\frac{1}{9}$ D $\frac{1}{10}$ (1)

1.5 The next number in the sequence
3 ; 6 ; 11 ; 18; . . . is



A 25 B 24
C 26 D 27 (1)

Get to know and understand the general term . . .

2.1 Write down the 1st 3 terms of a sequence if the general term is:

a) 3n b) 5n
c) 3n + 1 d) 5n – 2
e) n^2 f) n^3 (18)

2.2 Write down the 12th term for each case in Question 2.1. (6)

3. Use the table to answer the questions that follow:

x	1	2	3	4	**a**	10
y	3	6	9	12	21	**b**

3.1 Write down the relationship between *x* and y. (1)

3.2 Write down the values of a and b. (2)

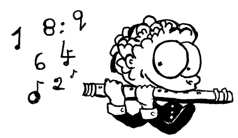
4. Study the given number sequence and answer the questions that follow:

3 ; 10 ; 17 ; 24 ; 31 ; . . .

4.1 Determine the constant difference between the consecutive terms in the number sequence. (1)

4.2 Write down the next two terms in the sequence. (2)

4.3 Write down the general term of the sequence. (2)



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5.1 Complete the table below:

Position in pattern	1	2	3	4	**5**
Term	1	8	27		

(2)

5.2 Write down the general term T_n of the above number pattern. (1)

5.3 If $T_n = 512$, determine the value of n. (2)

6.1 Write down the next TWO terms in the number sequence 7 ; 11 ; 15 ; . . . (2)

6.2 Write down the general term T_n of the above number sequence.

$T_n =$ (2)

6.3 Calculate the value of the 50^{th} term. (2)

7.1 Write down the next two terms in the given sequence:

3 ; 8 ; 13 ; ____ ; ____ (2)

7.2 Describe the pattern in Question 7.1 in your own words. (1)

7.3 Write down the general term of the given sequence in the form

$T_n =$ ____________________. (2)

7.4 Which term in the sequence is equal to 38? (3)

8.

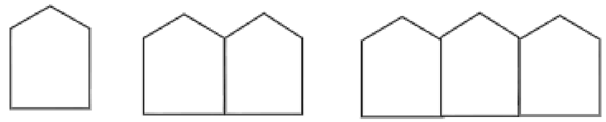

Figure 1 Figure 2 Figure 3

8.1 Study the above diagram pattern and complete the table.

Figure	1	2	3	4
Number of sides	5	9		

(2)

8.2 Describe the pattern in your own words. (1)

8.3 Write down the general term of the pattern in the form, $T_n =$ ______________________ (2)

9. Matchsticks are arranged as shown in the following figures:

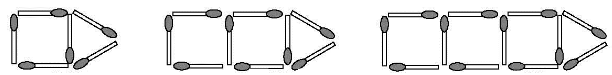

Figure 1 Figure 2 Figure 3

9.1 Determine the number of matchsticks in the next figure if the pattern is continued. (2)

9.2 Write down the general term of the given sequence of the matchsticks in the form

$T_n =$ ____________________. (2)

9.3 Determine the number of matchsticks in the 20^{th} figure. (2)

10. A tiler creates the following patterns with black and white tiles:

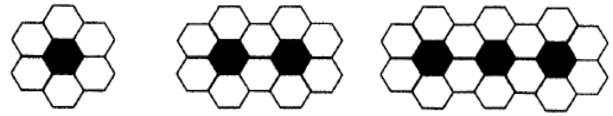

Figure 1 Figure 2 Figure 3

10.1 Study the above diagram pattern and complete the table.

Figure	1	2	3	4
Number of black tiles	1	2	3	4
Number of white tiles	6			

(2)

10.2 Write down the general term, T_n, of the number sequence created by the number of white tiles. (2)

11. Natural numbers are arranged as shown below.

$$1 + 2 = 3$$
$$4 + 5 + 6 = 7 + 8$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

Find the first number in the 20^{th} row if the pattern is continued another 17 times. (2)

For further practice in this topic – see The Answer Series Gr 9 Mathematics 2 in 1 on p. 1.15

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ALGEBRAIC EXPRESSIONS

(Solutions on page A3)

Terminology

1. Given the expression $2x - 7 - 8x^2$.

1.1 Write down the coefficient of x^2. (1)

1.2 Write down the constant term. (1)

1.3 Write the expression in descending powers of x. (1)

1.4 Write down the exponent in the term $2x$. (1)

1.5 Calculate the value of the expression $2x - 7 - 8x^2$ if $x = \frac{1}{2}$. (2)

2. Given the expression: $\frac{x - y}{3} + 4 - x^2$

Circle the letter of the **incorrect** statement.

A The expression consists of 3 terms.

B The coefficient of x is 1.

C The coefficient of x^2 is -1.

D The expression contains 2 variables. (1)

Substitution

3.1 Calculate the value of $2x^3 - 3x^2 + 9x + 2$ if $x = -2$. (4)

3.2 If $x = -1$, calculate the value of y if $y = 2x^2 - 3x + 5$. (2)

3.3 If $a = 2$, $b = -3$ and $c = \frac{1}{2}$, find the value of $\frac{5ac}{b}$. (4)

3.4 If $x = 2$ and $y = -3$, calculate the value of $3x^2 - 2xy - y^2$. (5)

Addition, Subtraction, Multiplication and Division

4. Answer the following questions.

4.1 Add $2b - 3a - c$ and $a - 4b + 2c$. (3)

4.2 Multiply $5x^2 - 3x$ by $-4x^2$. (2)

4.3 Divide $8a + 16a^2 - 4a^3$ by $2a$. (3)

4.4 Simplify $-3(x)(x) + 2x(-x)$. (3)

4.5 Multiply $4m - 3mn^5 + 2n$ by $-3m^2n$ (3)

4.6 Subtract $-2ab$ from $3ab$.

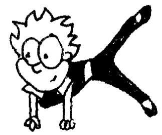
5. Simplify :

5.1 $(3x)^3 + 2x^3$ (2)

5.2 $(2x)^2 \times 3x^2$ (2)

5.3 $(a^2b^3)^2 . ab^2$ (2)

5.4 $2^5 - 1^5$ (2)



Fractions (+, −, ×, ÷)

5.5 $\frac{x}{2} + \frac{x}{5}$ (3)

5.6 $\frac{5a}{8} - \frac{5a}{12}$ (3)

5.7 $\frac{a^2b^2}{ac^2} \times \frac{4a^2bc}{20b^3}$ (2)

5.8 $\frac{6x^5}{x^4} - \frac{15x^3}{3x^2}$ (3)

5.9 $\frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4}$ (4)

5.10 $\frac{x-y}{y+x} \times \frac{(x+y)^2}{x-y}$ (2)

5.11 $\frac{x-2}{2x} - \frac{x-3}{3x}$ (5)

5.12 $\frac{4x^2}{2a^2} \div \frac{4x}{2a^2}$ (2)

5.13 $\frac{15x^2y^3 + 9x^2y^3}{8x^2y^3}$ (2)

5.14 $\frac{5a^2b}{3ab} \div \frac{20a^3b}{27}$ (5)

5.15 $\frac{5}{b} - \frac{4}{a} - \frac{a-b}{ab}$ (5)

5.16 $\frac{3a^{-2}b \times 24b^{-1}a^{-1}}{9a^{-4}b^{-3}}$ (3)

5.17 $\frac{x^2}{2} + \frac{2x^2}{3} - \frac{7x^2}{6}$ (3)

5.18 $\frac{6x^2}{7xy} \times \frac{3y^3}{2x}$ (2)

Square roots and cube roots

5.19 $\sqrt{225x^4} - \sqrt[3]{125x^6}$ (5)

5.20 $\sqrt{16x^{16} \times 25x^4}$ (3)

5.21 $\sqrt[3]{27x^{27}}$ (2)

5.22 $\sqrt{16a^2 + 9a^2}$ (2)

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Note: $x^2 = x \times x$ ... ***x multiplied by itself!***

STUDY THESE PRODUCTS OVER AND OVER AGAIN

So:

$$\mathbf{(x+y)^2} = (x+y)(x+y)$$
$$= x^2 + xy + xy + y^2$$
$$= x^2 \mathbf{+ 2xy} + y^2$$

So: $(x+y)^2$ **does not equal** $x^2 + y^2$

And:

$$\mathbf{(x-y)^2} = (x-y)(x-y)$$
$$= x^2 - xy - xy + y^2$$
$$= x^2 \mathbf{- 2xy} + y^2$$

So: $(x-y)^2$ **does not equal** $x^2 - y^2$

And, finally

$$\mathbf{(x+y)(x-y)} = x^2 - xy + xy - y^2$$
$$= \mathbf{x^2 - y^2}$$ ... *the difference of 2 squares!*

6. Determine the following products and simplify if necessary.

6.1 $4ab(5a^2b^2 + 2ab - 3)$ (3)

6.2 $3a^2bc^2(3a^2 - 4b - c)$ (3)

6.3 $(x+5)(x+2)$ (3)

6.4 $(x-2)(x-3)$ (3)

6.5 $(x+7)(x-1)$ (3)

6.6 $(2x-3)(x+1)$ (3)

6.7 $x(x+2) - (x-1)(x-3)$ (4)

6.8 $(x-3)^2 - x(x+4)$ (4)

6.9 $(2x-1)^2 - (x+1)(x-1)$ (4)

6.10 $2(x+2)^2 - (2x-1)(x+2)$ (4)

7. Complete the following products:

7.1 $(x+5)^2 = (x+5)(x+5) =$ ....

7.2 $(p-3)^2 = (p-3)(p-3) =$ ....

7.3 $(2a+3)^2 =$ ....

7.4 $(4x-1)^2 =$ ....

7.5 $(x+5)(x-5) =$ .... = ...

7.6 $(p-3)(p+3) =$ .... = ...

7.7 $(2a+3)(2a-3) =$ .... = ...

7.8 $(4x-1)(4x+1) =$ .... = ...

7.9 $(x+3)(x+4) =$ ....

7.10 $(x-3)(x-4) =$ ....

7.11 $(x+3)(x-4) =$ ....

7.12 $(x-3)(x+4) =$ ....

(24)

8.1 The value of $-x^2 - 2(2x-1)$ when $x = -2$ is ...

A 6 B 1

C -6 D -1 (2)

8.2 The LCM of $5a^3$ and $60a^2$ is ...

A $60a^5$ B $30a^3$

C $60a^3$ D $300a^6$ (2)

8.3 $\frac{x}{y} - 1 =$

A $\frac{y-x}{x}$ B $\frac{y-x}{y}$

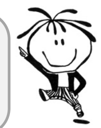
C $x - y$ D $\frac{x-y}{y}$ (2)

8.4 $\left(\frac{x}{3} - 3y\right)\left(\frac{x}{3} + 3y\right) =$

A $\frac{x^2}{9} + 3xy - 9y^2$ B $\frac{x^2}{9} + xy - 9y^2$

C $\frac{x^2}{9} + 9y^2$ D $\frac{x^2}{9} - 9y^2$ (2)

For further practice in this topic – see The Answer Series Gr 9 Mathematics 2 in 1 on pg 1.18 & 1.40



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FACTORISATION

(Solutions on page A7)

STUDY THIS TOPIC VERY WELL!

1. Common Factor

Always check for this first!

ab + ac = a(b + c)

BECAUSE: a(b + c) = ab + ac *. . . reversed*

Factorise:

NOTE

To factorise is to reverse a product!

1.1 $8p^3 + 4p^2$ (2)

1.2 $10t^2 - 5t$ (2)

1.3 $3x^2y - 9xy^2 + 12x^3y^3$ (2)

1.4 $2p^2 + 2$ (2)

1.5 $2(x + y) + a(x + y)$ (2)

1.6 $2(x + y) - t(x + y)$ (2)

✳1.7 $tx - ty - 2x + 2y$ (3)

✳ *a challenging question*

2. Difference between Squares

$$\mathbf{x^2 - y^2 = (x + y)(x - y)}$$

BECAUSE: $(x + y)(x - y) = x^2 - y^2$ *. . . reversed*

Factorise:

2 TERMS

2.1 $4x^2 - y^2$ (2)

2.2 $4x^2 - 4y^2$ (2)

2.3 $81 - 100a^2$ (2)

2.4 $9p^2 - 36q^2$ (3)

2.5 $7x^2 - 28$ (3)

Remember:

Always check for a Common Factor first!

3. Trinomials

3 TERMS

Complete the products:

▹ **Perfect Squares ➡ Perfect Square TRINOMIALS**

⬇ ⬇

$(x + 3)^2 = (x + 3)(x + 3) = x^2 \;.\,.\,.\,.\,.\,.\,.\; + 9 = x^2 \;.\,.\,.\,.\; + 9$

& $(x - 3)^2 = (x - 3)(x - 3) = x^2 \;.\,.\,.\,.\,.\,.\,.\; + 9 = x^2 \;.\,.\,.\,.\; + 9$

$\therefore\; x^2 + 6x + 9 = \;.\,.\,.\,.\,.\,.\,.$

& $x^2 - 6x + 9 = \;.\,.\,.\,.\,.\,.\,.$

$(a + b)^2 = (a + b)(a + b) = a^2 \;.\,.\,.\,.\,.\,.\,.\; + b^2 = a^2 \;.\,.\,.\,.\; + b^2$

& $(a - b)^2 = (a + b)(a + b) = a^2 \;.\,.\,.\,.\,.\,.\,.\; + b^2 = a^2 \;.\,.\,.\,.\; + b^2$

$\therefore\; a^2 + 2ab + b^2 = \;.\,.\,.\,.\,.\,.\,.$

& $a^2 - 2ab + b^2 = \;.\,.\,.\,.\,.\,.\,.$

▹ **Other products ➡ TRINOMIALS**

⬇ ⬇

$(x + 2)(x + 3)$ = ... = ...

$(x - 2)(x - 3)$ = ... = ...

$(x + 2)(x - 3)$ = ... = ...

$(x - 2)(x + 3)$ = ... = ...

Observe the results above to understand factorising trinomials

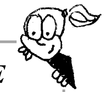
Factorise the following trinomials:

3.1 $a^2 + 8a + 16 = (a \;.\,.\,.\,.)(a \;.\,.\,.\,.) = (\qquad)^2$

3.2 $p^2 - 10p + 25 = (p \;.\,.\,.\,.)(p \;.\,.\,.\,.) = (\qquad)^2$

3.3 $x^2 + 5x + 6 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$

3.4 $x^2 - 5x + 6 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$



NOTE

To factorise is to reverse a product!

3.5 $x^2 + x - 6 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$

3.6 $x^2 - x - 6 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$

3.7 $x^2 - 11x + 18 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$

3.8 $x^2 + 11x + 18 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$

3.9 $x^2 - 7x - 18 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$

3.10 $x^2 + 7x - 18 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$

3.11 $x^2 + 9x + 18 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$

3.12 $x^2 - 9x + 18 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$

3.13 $x^2 + 3x - 18 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$

3.14 $x^2 - 3x - 18 = (x \;.\,.\,.\,.)(x \;.\,.\,.\,.)$

(28)

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Mixed Factorisation

Factorise fully :

4.1 $3a^3 - 9a^2 - 6a$ (3)

4.2 $2a^2 - 18a + 36$ (3)

4.3 $4(a + b) - x^2(a + b)$ (3)

4.4 $6x^3(a - b) + x(b - a)$ (4)

4.5 $6a^3 - 12a^2 + 18a$ (3)

- *Always first check for a common factor; then,*
- *make sure the factorisation is complete.*

Use factorisation to simplify the following fractions

5.1 $\dfrac{x^2 - 1}{3x + 3}$ (3)

5.2 $\dfrac{x^2 - 4x}{x^2 - 2x - 8}$ (3)

5.3 $\dfrac{3a - 6b}{4b - 2a}$ (3)

5.4 $\dfrac{2x^2 - 8}{3x - 12} \times \dfrac{x^2 - 4x}{x - 2}$ (4)

5.5 $\dfrac{x^2 + 2x}{x^3 - 2x} \div \dfrac{x^2 - 4}{x - 2}$ (5)

For further practice in this topic – see The Answer Series Gr 9 Mathematics 2 in 1 on pg 1.40

NOTES

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ALGEBRAIC EQUATIONS
(Linear and Quadratic)

(Solutions on page A9)

1.1 If 3 is a root of the equation $x^2 + x + t = 0$, the value of t is . . .

A 12

B −12

C $\frac{1}{2}$

D $-\frac{1}{2}$ (1)

A ***'root'*** *of an equation is* ***'the solution'*** *of the equation.*

1.2 Calculate the value of p if 2p + 12 = 58.

A 22

B 12

C 18

D 23 (1)

1.3 If $(x-1)(x+2) = 0$ then x =

A −1 or 0

B 1 or −2

C 1

D −2 (1)

1.4 If $\frac{3x}{2} = -6$ then x =

A 9

B 4

C −9

D −4 (1)

1.5 The product of a number and 6 decreased by 4 is equal to 20. Which one of the following equations matches the statement?

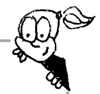
A $6x + 4 = 20$

B $6x - 4 = 20$

C $6(x + 4) = 20$

D $6 - 4x = 20$ (1)

2. Solve for x in the following LINEAR equations (i.e. find the value of x which makes the equation true).



These examples can be done by inspection.

2.1 $x + 5 = 2$ (1)

2.2 $x - 3 = -4$ (1)

2.3 $2x = 12$ (1)

2.4 $\frac{x}{5} = 6$ (1)

3. Solve for x:

3.1 $3x - 1 = 5$ (2)

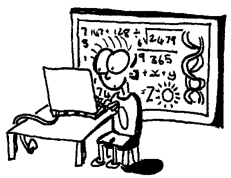
3.2 $2(x + 1) = 10$ (3)

3.3 $8x + 3 = 3x - 22$ (2)

3.4 $3(x + 6) = 12$ (2)

3.5 $2x - 5 = 5x + 16$ (3)

3.6 $x^3 + x^3 = 2$ (2)



Equations including fractions

4. Solve for x:

4.1 $\frac{x-2}{4} + \frac{2x+1}{3} = \frac{5}{3}$ (5)

4.2 $\frac{x+2}{3} - \frac{x-3}{4} = 0$ (3)

4.3 $\frac{2x-3}{2} - \frac{x+1}{3} = \frac{3x-1}{2}$ (4)

4.4 $x - \frac{x-1}{2} = 3$ (4)

4.5 $\frac{x+1}{3} - \frac{x-1}{6} = 1$ (3)

Quadratic Equations

5. Solve for x:

5.1 $(x-3)(x+4) = 0$ (2)

5.2 $x^2 - 5x - 6 = 0$ (3)

5.3 $x^2 - 1 = 0$ (3)

5.4 $x^2 - 2x = 0$ (3)

6. Solve for x:

6.1 $2(x-2)^2 = (2x-1)(x-3)$ (4)

6.2 $(x-2)^2 + 3x - 2 = (x+3)^2$ (4)

6.3 $(x-3)^2 = 16$ (6)

Other . . .

7. Solve for x:

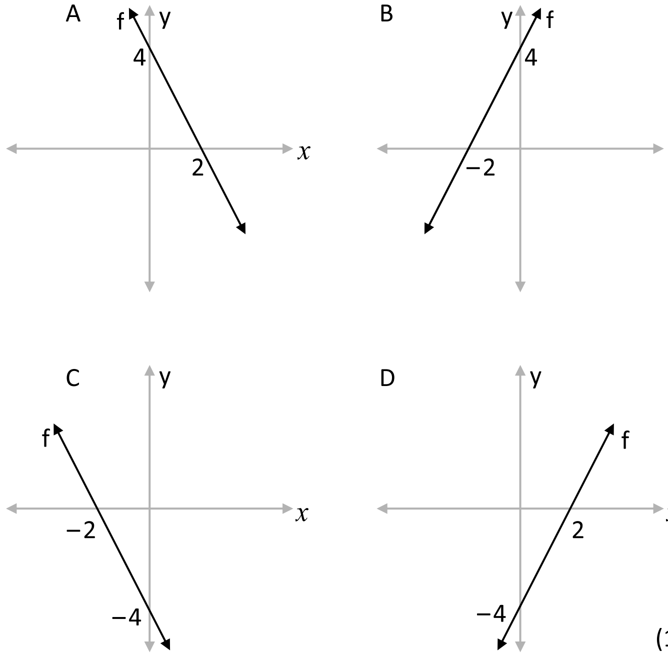
7.1 $\sqrt{\sqrt{\sqrt{x}}} = 2$ 7.2 $\sqrt{\frac{1}{\sqrt{x}}} = 2$ (3)(3)

For further practice in this topic – see The Answer Series Gr 9 Mathematics 2 in 1 on pg 1.21 & 1.42

GRAPHS

(Solutions on page A13)

1.1 The graph of the straight line defined by $f(x) = 2x + 4$ is



(1)

1.2 If T is a point on the line defined by $y = x$, the coordinates of T are . . .

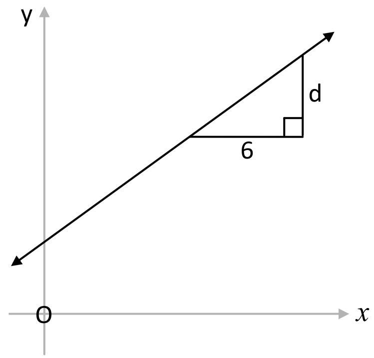
A (5; −5)

B (5; 0)

C (−5; 5)

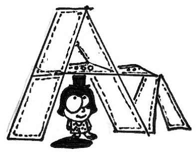
D (−5; −5) (1)

1.3



The gradient of the line shown above is $\frac{2}{3}$.

What is the value of d ?



A 3

B 4

C 6

D 9 (1)

1.4 What is the y–intercept of the graph defined by $4x + 2y = 12$?

A −4

B −2

C 6

D 12 (1)

1.5 The straight line graph defined by $3y + 2x + 1 = 0$ will cut the x–axis at the point ...

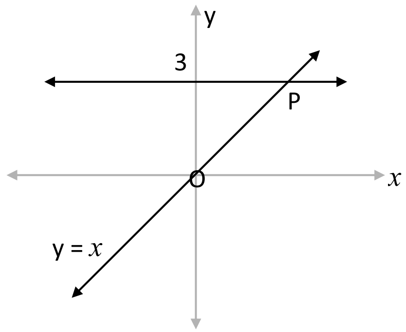
A $(-2\,;0)$

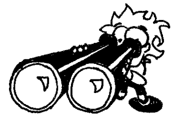
B $(-\frac{1}{2}\,;0)$

C $(-3\,;0)$

D $(-\frac{1}{3}\,;0)$ (1)

2. Determine the co–ordinates of P in the graph below.





(1)

3. Use the given equation to complete each of the following tables.

3.1 $y = 3x - 5$

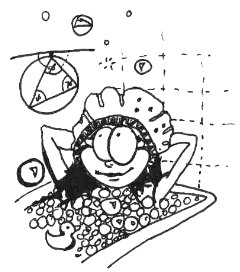
x	−2	−1	0	1
y				

(2)

3.2 $y = -\frac{2}{3}x - 1$

x	−3	−1	0	1
y				

(2)



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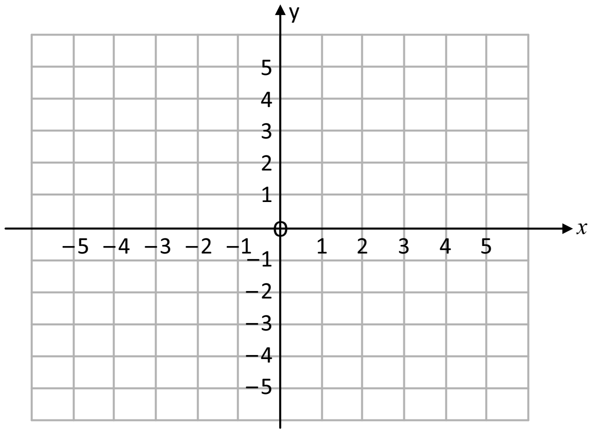
4.1 On the given grid draw the graphs defined by $y = 3x - 2$ and $y = 3x + 1$ on the same set of axes.

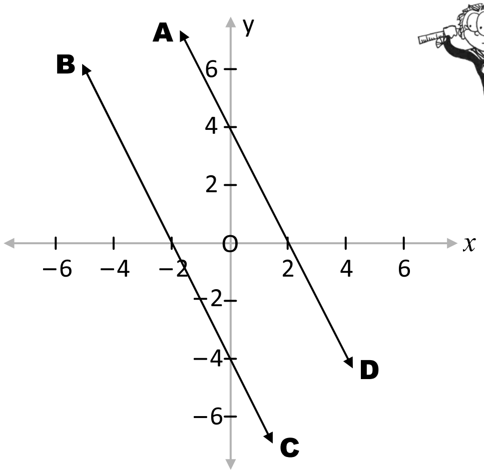
Label each graph and clearly mark the points where the graphs cut the axes.



(6)

4.2 What is the relationship between the lines that you have drawn? (1)

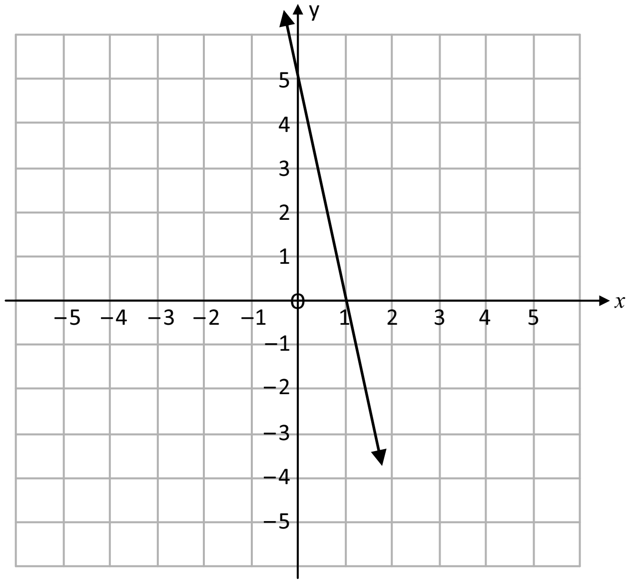
5.1 Write down the defining equation of each of the following straight line graphs.



(4)

5.2 What can you deduce about lines **AD** and **BC**? Give a reason for your answer. (2)

6. Study the graph below.

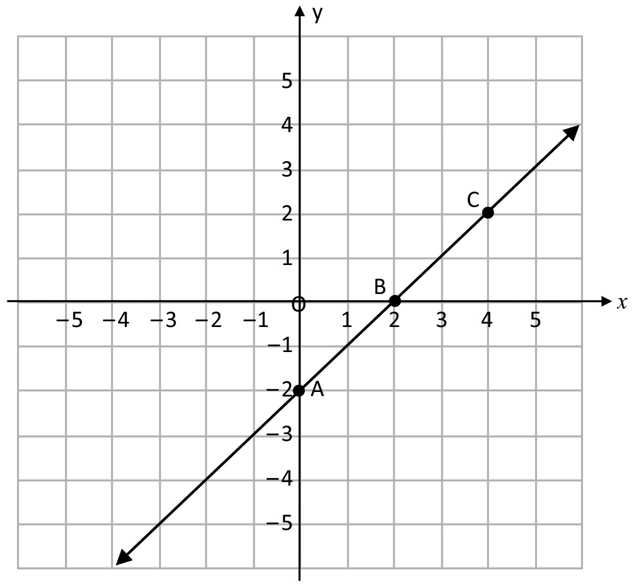


6.1 Use the graph to calculate the gradient of the straight line. (3)

6.2 Determine the equation of the straight line. (2)

6.3 Write down the gradient of any other straight line which can be drawn parallel to the given line. (1)

7. Use the graph below to answer the questions that follow.

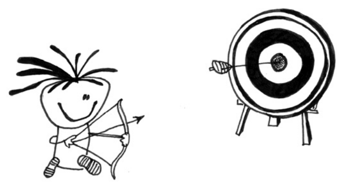


7.1 Write down the coordinates of points A, B and C in the table.

	A	**B**	**C**
x–coordinate			
y–coordinate			

(3)

7.2 Use the table in Question 7.1 or any other method to determine the equation of line ABC. (2)



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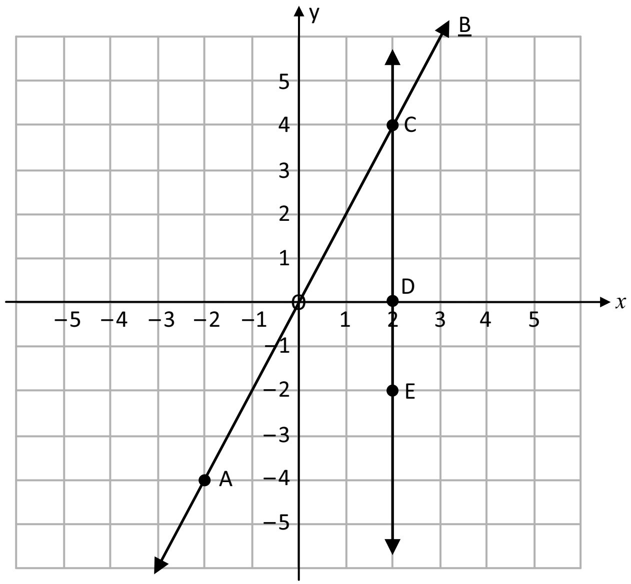
8. Study the straight line graphs below and answer the questions that follow.



Complete:

8.1 The equation of the line CD is . . . . (1)

8.2 The equation of the line AB is. . . . . (2)

8.3 If DE = 2, the co–ordinates of E are. . . . . (2)

8.4 The length of CE is . . . . (1)

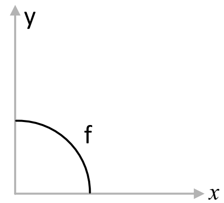
9. Underline the word, the number or the equation between brackets so that each of the following statements is correct.

9.1 The lines $x = 4$ and $x = -4$ are (parallel / perpendicular) to one another. (1)

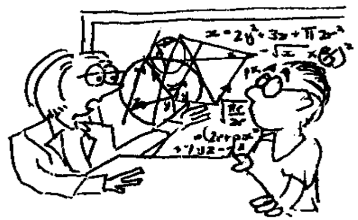
9.2 The equation of the horizontal line through the point P(3; −2) is ($x = 3$ / $y = -2$). (1)

9.3 The gradient of the line defined by $y - 4x + 5 = 0$ is equal to (−4 / 4). (1)

9.4 This graph of f below represents a (linear / non–linear) function.



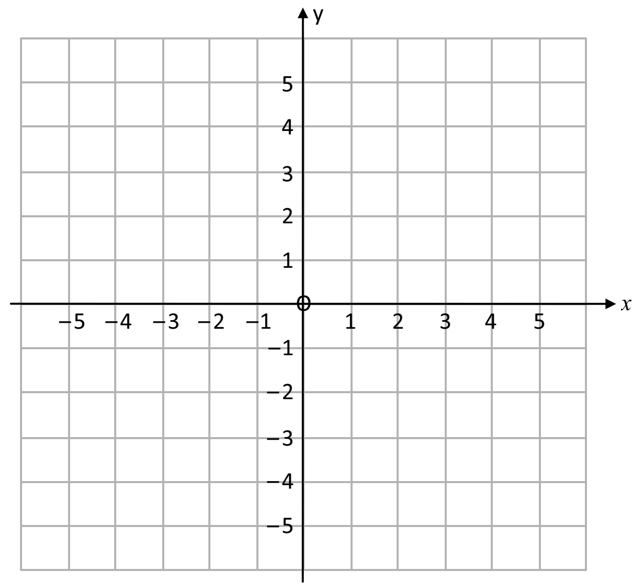
(1)



10.1 On the given grid draw the graphs defined by

$y = -\frac{2}{3}x + 1$ and $y = \frac{3}{2}x - 1$.

Label each graph and clearly mark the points where each graph cuts the x–axis and the y–axis. (6)



10.2 What is the relationship between the lines that you have drawn? (1)

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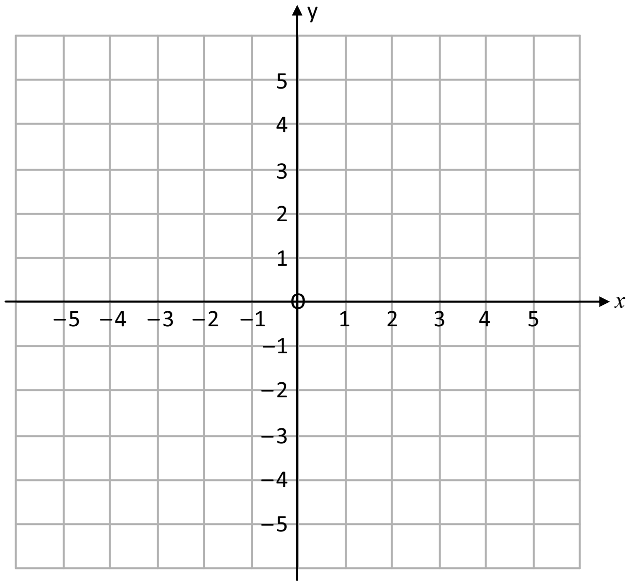
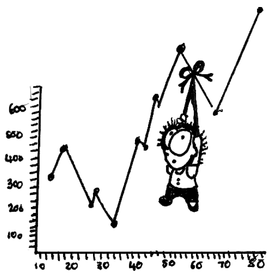
11. Use the grid below to answer the questions that follow.

11.1 Draw the graphs defined by $y = -2x + 4$ and $x = 1$ on the given set of axes. Label each graph and clearly mark the points where the lines cut the axes.



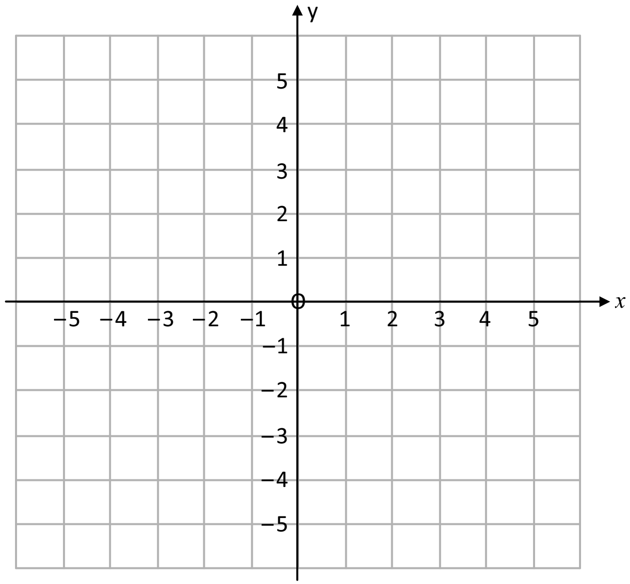
(6)

11.2 Write down the coordinates of the point where the two lines cut one another. (2)



12.1 On the same set of axes, draw and label the graphs defined by $y = -2x + 1$ and $y = x - 2$.

Use the given grid and clearly indicate the points where the lines cut the axes.



(8)

12.2 The lines intersect at T.

Show by calculation that the coordinates of T are $x = 1$ and $y = -1$ or $(1; -1)$. (2)

For further practice in this topic – see The Answer Series Gr 9 Mathematics 2 in 1 on pg 1.44

NOTES

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Gr 9 Maths: Content Area 2
Patterns, Algebra & Graphs
ANSWERS

- Patterns
- Algebraic Expressions
- Factorisation
- Algebraic Equations
- Graphs

PATTERNS

1.1 **C** ◄ . . . 1^2 ; 3^2 ; 5^2 ; $\mathbf{7^2}$

1.2 **C** ◄ . . . 18 ; 36 ; **54** ; 72 ; **90** (+18 +18 +18 +18)

1.3 **A** ◄ . . . $\frac{1}{2^0}$; $\frac{1}{2^1}$; $\frac{1}{2^2}$; $\mathbf{\frac{1}{2^3}}$; $\frac{1}{2^4}$

1.4 **A** ◄ . . . $\frac{1}{3}$; $\mathbf{\frac{1}{6}}$; $\frac{1}{12}$; $\frac{1}{24}$; $\frac{1}{48}$

1.5 **D** ◄ . . . 1^2+2 ; 2^2+2 ; 3^2+2 ; 4^2+2 ; $\mathbf{5^2+2}$

2.1 a) $T_n = 3n$:

3 ; 6 ; 9 ◄ . . . 3(**1**) ; 3(**2**) ; 3(**3**)

b) $T_n = 5n$:

5 ; 10 ; 15 ◄ . . . 5(**1**) ; 5(**2**) ; 5(**3**)

c) $T_n = 3n + 1$:

4 ; 7 ; 10 ◄ . . . 3(**1**) + 1 ; 3(**2**) + 1 ; 3(**3**) + 1

d) $T_n = 5n - 2$:

3 ; 8 ; 13 ◄ . . . 5(**1**) – 2 ; 5(**2**) – 2 ; 5(**3**) – 2

e) $T_n = n^2$:

1 ; 4 ; 9 ◄ . . . $(\mathbf{1})^2$; $(\mathbf{2})^2$; $(\mathbf{3})^2$

f) $T_n = n^3$:

1 ; 8 ; 27 ◄ . . . $(\mathbf{1})^3$; $(\mathbf{2})^3$; $(\mathbf{3})^3$

2.2 a) $T_{12} = 3(\mathbf{12}) = \mathbf{36}$ ◄

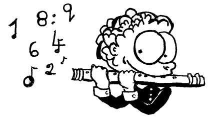
b) $T_{12} = 5(\mathbf{12}) = \mathbf{60}$ ◄

c) $T_{12} = 3(\mathbf{12}) + 1 = \mathbf{37}$ ◄

d) $T_{12} = 5(\mathbf{12}) - 2 = \mathbf{58}$ ◄

e) $T_{12} = (\mathbf{12})^2 = \mathbf{144}$ ◄

f) $T_{12} = (\mathbf{12})^3 = \mathbf{1\ 728}$ ◄



3.1 **y = 3*x*** ◄

3.2 **a = 7** ◄ . . . **7** × 3 = 21

b = 30 ◄ . . . 10 × 3 = **30**

4.1 The constant difference = **7** ◄

4.2 3 ; 10 ; 17 ; 24 ; 31 ; **38** ; **45** ◄

4.3 The constant **difference** is **7** . . . *see Question 4.1*

So, write down **the multiples of 7** . . . where $T_n = 7n$:

7 ; 14 ; 21 ; 28 ; 35 ; . . .

and compare the given sequence :

3 ; 10 ; 17 ; 24 ; 31 ; . . .

Each term is **4 less** than the multiples of 7.

$\therefore\ \mathbf{T_n = 7n - 4}$ ◄

5.1

Position in pattern	1	2	3	4	5
Term	1	8	27	**64**	**125**

◄

5.2 $\mathbf{T_n = n^3}$ ◄

5.3 If $T_n = 512$, then **n = 8** ◄

. . . $512 = 2^9$

$= 2^3 \times 2^3 \times 2^3$

$= 8 \times 8 \times 8$

$= \mathbf{8^3}$ ◄

2 | 512
2 | 256
2 | 128
2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
2

6.1 7 ; 11 ; 15 ; **19** ; **23** ◄

6.2 The common **difference** is **4**

So, compare the **multiples of 4** . . . where $T_n = 4n$:

4 ; 8 ; 12 ; 16 ; . . .

to the given sequence :

7 ; 11 ; 15 ; 19 ; . . .

Each term is **3 more** than the multiples of 4.

$\therefore\ \mathbf{T_n = 4n + 3}$ ◄ . . . 7 ; 11 ; 15 ; 19 ; 23 (+4 +4 +4 +4)

6.3 $T_{50} = 4(50) + 3$

$= \mathbf{203}$ ◄

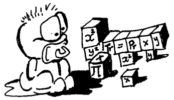
7.1 3 ; 8 ; 13 ; **18** ; **23** ◄

7.2 Each term is 5 more than the previous term ◄

7.3 Compare $T_n = 5n$: 5 ; 10 ; 15 ; . . .
to : 3 ; 8 ; 13 ; . . .

Each term is **2 less** than the multiples of 5

$\therefore$ **$T_n = 5n - 2$** ◄



7.4 38 is 2 less than 40 ; 40 = 5 **x 8**

$\therefore$ 38 is the **8th** term ◄

OR : Solve the equation :

$5n - 2 = 38$ . . . *the n^{th} term = 38*

Add 2: $\therefore\ 5n = 40$

Divide by 5: $n = 8$

$\therefore$ The **8th** term ◄

8.1

Figure	1	2	3	4
Number of sides	5	9	**13**	**17**

◄

8.2 Each figure has four more sides than the previous figure ◄

8.3 **$T_n = 4n + 1$** ◄

+4 +4 +4
. . . 5 ; 9 ; 13 ; 17

*& each term is **1 more** than the multiples of 4*

9.

Figure	1	2	3
Number of matchsticks	6	9	12

9.1 Number of matchsticks in Figure 4 = **15** ◄

9.2 **$T_n = 3n + 3$** ◄

+3 +3
. . . 6 ; 9 ; 12

*& each term is **3 more** than the multiples of 3*

9.3 $T_{20} = 3(\mathbf{20}) + 3$
= **63 matchsticks** ◄

10.1

Figure	1	2	3	4
Number of black tiles	1	2	3	4
Number of white tiles	6	**10**	**14**	**18**

◄

10.2 **$T_n = 4n + 2$** ◄

+4 +4 +4
. . . 6 ; 10 ; 14 ; 18

*& each term is **2 more** than the multiples of 4*

---

11. Look at the pattern formed by the first numbers of each line :

1 ; 4 ; 9 ; . . . the squares !

row **1** row **2** row **3**
($\mathbf{1}^2$) ($\mathbf{2}^2$) ($\mathbf{3}^2$)

$\therefore$ The first number in the **20th** row will be $\mathbf{20}^2$ = **400** ◄

NOTES

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ALGEBRAIC EXPRESSIONS

Terminology

1.1 **−8** ◄ . . . $\mathbf{-8}x^2$

1.2 **−7** ◄ . . . *the term with no variable*

1.3 $\mathbf{-8x^2 + 2x - 7}$ ◄

1.4 **1** ◄ . . . $2x^1$

1.5 $2x - 7 - 8x^2$

$x = \frac{1}{2}$: $\quad 2\left(\frac{1}{2}\right) - 7 - 8\left(\frac{1}{2}\right)^2$

$= \left(\frac{2}{1}\right)\left(\frac{1}{2}\right) - 7 - \left(\frac{8}{1}\right)\left(\frac{1}{4}\right)$

$= 1 - 7 - 2$

$= \mathbf{-8}$ ◄

---

2. **B** ◄ . . . *x is part of the fraction $\frac{x - y}{3}$, which can also be written as $\frac{1}{3}(x - y) = \frac{1}{3}x - \frac{1}{3}y$.*

$\therefore$ The coefficient of x is $\frac{1}{3}$.

The two variables are x and y.

Substitution

3.1 $2x^3 - 3x^2 + 9x + 2$

$\underline{x = -2}$: $\quad 2(-2)^3 - 3(-2)^2 + 9(-2) + 2$

$= 2(-8) - 3(4) + 9(-2) + 2$

$= -16 - 12 - 18 + 2$

$= \mathbf{-44}$ ◄

3.2 $y = 2x^2 - 3x + 5$

$\underline{x = -1}$: $\quad y = 2(-1)^2 - 3(-1) + 5$

$= 2 + 3 + 5$

$= \mathbf{10}$ ◄

3.3 $\frac{5ac}{b}$

$= \frac{\frac{5}{1}\left(\frac{2}{1}\right)\left(\frac{1}{2}\right)}{(-3)}$ . . . *it is useful to put whole numbers over 1 when fractions are involved*

$= \frac{5(1)}{-3}$

$= -\mathbf{\frac{5}{3}}$ ◄

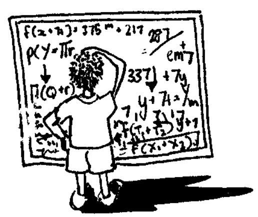
3.4 $3x^2 - 2xy - y^2$

$= 3(2)^2 - 2(2)(-3) - (-3)^2$

$= 3(4) - 2(-6) - (9)$

$= 12 + 12 - 9$

$= \mathbf{15}$ ◄



Addition, Subtraction, Multiplication and Division

4.1 $(2b - 3a - c) + (a - 4b + 2c)$

$= 2b - 3a - c + a - 4b + 2c$

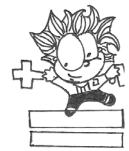
$= -3a + a + 2b - 4b - c + 2c$

$= \mathbf{-2a - 2b + c}$ ◄

OR: $\quad -3a + 2b - c$
Add $\quad \underline{a - 4b + 2c}$
$\quad -2a - 2b + c$ ◄

4.2 $-4x^2(5x^2 - 3x)$

$= \mathbf{-20x^4 + 12x^3}$ ◄



Distributive Property :
$a(b + c) = ab + ac$

4.3 $\frac{8a + 16a^2 - 4a^3}{2a}$

$= \frac{8a}{2a} + \frac{16a^2}{2a} - \frac{4a^3}{2a}$ . . . *Each **term** in the numerator must be divided by 2a.*

$= \mathbf{4 + 8a - 2a^2}$ ◄

4.4 $-3(x)(x) + 2x(-x)$

$= -3x^2 - 2x^2$ . . . *LIKE TERMS* ☺

$= \mathbf{-5x^2}$ ◄

4.5 $-3m^2n(4m - 3mn^5 + 2n)$ . . . *Distributive Property*

$= \mathbf{-12m^3n + 9m^3n^6 - 6m^2n^2}$ ◄

4.6 $3ab - (-2ab)$

$= 3ab + 2ab$ . . . *LIKE TERMS* ☺

$= \mathbf{5ab}$ ◄

5.1 $(3x)^3 + 2x^3$ . . . *$(3x)^3 = 3^3 . x^3$*

$= 27x^3 + 2x^3$ . . . *LIKE TERMS* ☺

$= \mathbf{29x^3}$ ➤

5.2 $(2x)^2 \times 3x^2$ . . . *$(2x)^2 = 2x \times 2x$*

$= 4x^2 \times 3x^2$

$= \mathbf{12x^4}$ ➤

5.3 $(a^2b^3)^2 . ab^2$

$= a^4b^6 . ab^2$

$= \mathbf{a^5b^8}$ ➤

5.4 $2^5 - 1^5$ . . . *$2^5 = 2 \times 2 \times 2 \times 2 \times 2$*

$= 32 - 1$ *$1^5 = 1 \times 1 \times 1 \times 1 \times 1$*

$= \mathbf{31}$ ➤

Fractions (+, −, ×, ÷)

5.5 $\frac{x \times 5}{2 \times 5} + \frac{x \times 2}{5 \times 2}$

$= \frac{5x + 2x}{10}$

$= \mathbf{\frac{7x}{10}}$ ➤

When we add or subtract fractions, we must determine a common denominator

5.6 $\frac{5a \times 3}{8 \times 3} - \frac{5a \times 2}{12 \times 2}$

$= \frac{15a - 10a}{24}$

$= \mathbf{\frac{5a}{24}}$ ➤

NB : *'Keep' the denominator ! Do not multiply by it.*

5.7 $\frac{a^2b^2}{ac^2} \times \frac{4a^2bc}{20b^3}$

When we multiply fractions, we may cancel FACTORS.

$= \frac{ab^2}{c^2} \times \frac{a^2c}{5b^2}$

$= \mathbf{\frac{a^3}{5c}}$

NB : Understand the expression

$\frac{a \times a \times b \times b}{a \times c \times c} \times \frac{4 \times a \times a \times b \times c}{20 \times b \times b \times b}$

. . . and then cancel !

5.8 $\frac{6x^5}{x^4} - \frac{15x^3}{3x^2}$

$= 6x - 5x$ . . . *LIKE TERMS* ☺

$= \mathbf{x}$ ➤

NB : Understand the expression

$\frac{6 \times x \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}} - \frac{{}^{5}\cancel{15} \times x \times \cancel{x} \times \cancel{x}}{\cancel{3} \times \cancel{x} \times \cancel{x}}$

5.9 $\frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4}$

$= \frac{2x + 1 - 2(x + 2) - 1}{4}$

$= \frac{2x + 1 - 2x - 4 - 1}{4}$

$= \frac{-4}{4}$

$= \mathbf{-1}$ ➤

This is an ***expression****:*

keep the value the same ; do ***not*** *multiply it !*

All terms need to be written over a common (the same) denominator.

5.10 $\frac{x - y}{y + x} \times \frac{(x + y)^2}{x - y}$

$= \frac{\cancel{(x - y)}}{\cancel{(x + y)}} \times \frac{(x + y)^{\cancel{2}}}{\cancel{(x - y)}}$

$= \frac{x + y}{1}$

$= \mathbf{x + y}$ ➤

Compare to : $\frac{a}{b} \times \frac{b^2}{a}$

$= b$ ➤

5.11 $\frac{x - 2}{2x} - \frac{x - 3}{3x}$ . . . *Do not multiply !*

$= \frac{3(x - 2) - 2(x - 3)}{6x}$ . . . *NB : Brackets !*

$= \frac{3x - 6 - 2x + 6}{6x}$ . . . *Keep the denominator, 6x !*

$= \frac{x}{6x}$

$= \mathbf{\frac{1}{6}}$ ➤

NB :

It is only when working with fractions **IN EQUATIONS** (Page Q6, Question 4), that you **logically** multiply both sides of the equation !

5.12 $\frac{4x^2}{2a^2} \div \frac{4x}{2a^2}$

$= \frac{4x^2}{2a^2} \times \frac{2a^2}{4x}$

$= \mathbf{x}$ ➤

5.13 $\frac{15x^2y^3 + 9x^2y^3}{8x^2y^3}$ . . . *LIKE TERMS* ☺

$= \frac{24x^2y^3}{8x^2y^3}$

$= \mathbf{3}$ ➤

5.14 $\frac{5a^2b}{3ab} \div \frac{20a^3b}{27}$

$= \frac{5a}{3} \times \frac{27}{20a^3b}$

$= \mathbf{\frac{9}{4a^2b}}$ ➤

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5.15 $\frac{5}{b} - \frac{4}{a} - \frac{a-b}{ab}$ ... *Do not multiply!*

$= \frac{5a - 4b - (a - b)}{ab}$ ... *NB: Brackets!*

$= \frac{5a - 4b - a + b}{ab}$ ... *Keep the denominator, ab!*

$= \mathbf{\frac{4a - 3b}{ab}}$

5.16 $\frac{3a^{-2}b \times 24b^{-1}a^{-1}}{9a^{-4}b^{-3}}$

$= \frac{72a^{-3}}{9a^{-4}b^{-3}}$

$= \mathbf{8ab^3}$ ... $\frac{a^{-3}}{a^{-4}} = a^{-3-(-4)} = a^1$

& $\frac{1}{b^{-3}} = b^3$

5.17 $\frac{x^2}{2} + \frac{2x^2}{3} - \frac{7x^2}{6}$

$= \frac{3x^2 + 2(2x^2) - 7x^2}{6}$ ... *Write all 3 terms over a common denominator, 6.*

$= \frac{3x^2 + 4x^2 - 7x^2}{6}$ ... *Do not multiply (by 6)!*

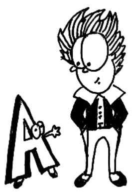
$= \frac{0}{6}$

$= \mathbf{0}$

5.18 $\frac{6x^2}{7xy} \times \frac{3y^3}{2x}$

$= \frac{6x}{7y} \times \frac{3y^3}{2x}$

$= \mathbf{\frac{9y^2}{7}}$



Square roots and cube roots

5.19 $\sqrt{225x^4} - \sqrt[3]{125x^6}$

$= 15x^2 - 5x^2$

$= \mathbf{10x^2}$

$$\begin{array}{r|l} 3 & 225 \\ 3 & 75 \\ 5 & 25 \\ & 5 \end{array} \qquad \begin{array}{r|l} 5 & 125 \\ 5 & 25 \\ & 5 \end{array}$$

$\therefore\ 125 = 5^3$

$\therefore\ 225 = 3^2 \times 5^2$ $\qquad \therefore\ \sqrt[3]{125} = 5$

$\therefore\ \sqrt{225} = 3 \times 5$

5.20 $\sqrt{16x^{16} \times 25x^4}$

$= \sqrt{16x^{16}} \,.\, \sqrt{25x^4}$

$= 4x^8 \,.\, 5x^2$

$= \mathbf{20x^{10}}$

OR: $= \sqrt{400x^{20}}$

$= \sqrt{20x^{10}}$

5.21 $\sqrt[3]{27x^{27}}$ ... $3 \times 3 \times 3 = 27$; $x^9 \times x^9 \times x^9 = x^{27}$

$= \mathbf{3x^9}$

5.22 $\sqrt{16a^2 + 9a^2}$ ... *1st add LIKE TERMS*

$= \sqrt{25a^2}$

$= \mathbf{5a}$

NB: $\sqrt{16a^2 + 9a^2} \neq 4a + 3a$!

6.1 $4ab(5a^2b^2 + 2ab - 3)$

$= \mathbf{20a^3b^3 + 8a^2b^2 - 12ab}$

6.2 $3a^2bc^2(3a^2 - 4b - c)$

$= \mathbf{9a^4bc^2 - 12a^2b^2c^2 - 3a^2bc^3}$

Distributive Property:

$a(b + c)$
$= ab + ac$

and

$a(b - c)$
$= ab - ac$

6.3 $(x + 5)(x + 2) = x^2 + 2x + 5x + 10$

$= x^2 + 7x + 10$

6.4 $(x - 2)(x - 3) = x^2 - 3x - 2x + 6$

$= x^2 - 5x + 6$

6.5 $(x + 7)(x - 1) = x^2 - x + 7x - 7$

$= x^2 + 6x - 7$

6.6 $(2x - 3)(x + 1)$

$= 2x^2 + 2x - 3x - 3$

$= \mathbf{2x^2 - x - 3}$

6.7 $x(x + 2) - (x - 1)(x - 3)$

$= x^2 + 2x - (x^2 - 3x - x + 3)$

$= x^2 + 2x - (x^2 - 4x + 3)$

$= x^2 + 2x - x^2 + 4x - 3$

$= \mathbf{6x - 3}$

6.8 $(x - 3)^2 - x(x + 4)$

$= x^2 - 6x + 9 - x^2 - 4x$

$= \mathbf{-10x + 9}$

$(x - 3)^2$

$= (x - 3)(x - 3)$

$= x^2 \mathbf{- 3x - 3x} + 9$

$= x^2 \mathbf{- 6x} + 9$

Perfect Square Trinomial

6.9 $(2x-1)^2 - (x+1)(x-1)$

$= (2x-1)(2x-1) - (x^2-1)$

$= 4x^2 - 4x + 1 - x^2 + 1$

$= \mathbf{3x^2 - 4x + 2}$ ➤

Perfect Square Trinomial ➡

$(2x-1)^2 = (2x-1)(2x-1)$

$= 4x^2 \mathbf{- 2x - 2x} + 1$

$= 4x^2 \mathbf{- 4x} + 1$

Difference between Squares ➡

& $(x+1)(x-1) = x^2 - x + x - 1$

$= x^2 - 1$

6.10 $2(x+2)^2 - (2x-1)(x+2)$

$= 2(x+2)(x+2) - (2x^2 + 4x - x - 2)$

$= 2(x^2 + 4x + 4) - (2x^2 + 3x - 2)$

$= 2x^2 + 8x + 8 - 2x^2 - 3x + 2$

$= \mathbf{5x + 10}$ ➤

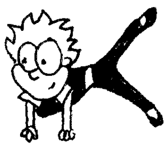
Perfect Square Trinomial ➡

$(x+2)^2$

$= (x+2)(x+2)$

$= x^2 \mathbf{+ 2x + 2x} + 4$

$= x^2 \mathbf{+ 4x} + 4$



7.1 ➡ 7.4: Perfect Square Trinomials

7.1 $(x+5)^2 = (x+5)(x+5)$

$= x^2 \mathbf{+ 5x + 5x} + 25$

$= \mathbf{x^2 + 10x + 25}$ ➤

7.2 $(p-3)^2 = (p-3)(p-3)$

$= p^2 \mathbf{- 3p - 3p} + 9$

$= \mathbf{p^2 - 6p + 9}$ ➤

7.3 $(2a+3)^2 = (2a+3)(2a+3)$

$= 4a^2 \mathbf{+ 6a + 6a} + 9$

$= \mathbf{4a^2 + 12a + 9}$ ➤

7.4 $(4x-1)^2 = (4x-1)(4x-1)$

$= 16x^2 \mathbf{- 4x - 4x} + 1$

$= \mathbf{16x^2 - 8x + 1}$ ➤

7.5 ➡ 7.8: Difference between Squares

7.5 $(x+5)(x-5) = \mathbf{x^2 - 5x + 5x - 25} = \mathbf{x^2 - 25}$

7.6 $(p-3)(p+3) = \mathbf{p^2 + 3p - 3p - 9} = \mathbf{p^2 - 9}$

7.7 $(2a+3)(2a-3) = \mathbf{4a^2 - 6a + 6a - 9} = \mathbf{4a^2 - 9}$

7.8 $(4x-1)(4x+1) = \mathbf{16x^2 + 4x - 4x - 1} = \mathbf{16x^2 - 1}$

7.9 ➡ 7.12: Observe **how** the **trinomial** is obtained in each case.

7.9 $(x+3)(x+4) = x^2 + 4x + 3x + 12 = \mathbf{x^2 + 7x + 12}$ ➤

7.10 $(x-3)(x-4) = x^2 - 4x - 3x + 12 = \mathbf{x^2 - 7x + 12}$ ➤

7.11 $(x+3)(x-4) = x^2 - 4x + 3x - 12 = \mathbf{x^2 - x - 12}$ ➤

7.12 $(x-3)(x+4) = x^2 + 4x - 3x - 12 = \mathbf{x^2 + x - 12}$ ➤

8.1 **B** ➤ . . . $-(-2)^2 - (2(-2) - 1)$

$= -(4) - (-4 - 1)$

$= -4 - (-5)$

$= -4 + 5$

$= 1$

8.2 **C** ➤

8.3 **D** ➤ . . . $\dfrac{x}{y} - \dfrac{1^{\times y}}{1_{\times y}} = \dfrac{x - y}{y}$ . . . *Write the terms over the same denominator.*

8.4 **D** ➤ . . . $\left(\dfrac{x}{3} - 3y\right)\left(\dfrac{x}{3} + 3y\right)$

$= \left(\dfrac{x}{3}\right)^2 - (3y)^2$ . . . *difference of squares*

$= \dfrac{x^2}{9} - 9y^2$

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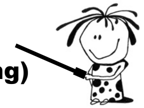
FACTORISATION

1. Common Factor



Check each answer by multiplying back (to the beginning)

1.1 $8p^3 + 4p^2$
$= \mathbf{4p^2(2p + 1)}$ ◄

1.2 $10t^2 - 5t$
$= \mathbf{5t(2t - 1)}$ ◄

1.3 $3x^2y - 9xy^2 + 12x^3y^3$
$= \mathbf{3xy(x - 3y + 4x^2y^2)}$ ◄

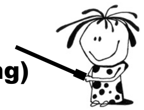
1.4 $2p^2 + 2$
$= \mathbf{2(p^2 + 1)}$ ◄

1.5 $2(x + y) + a(x + y)$
$= \mathbf{(x + y)(2 + a)}$ ◄

1.6 $2(x + y) - t(x + y)$
$= \mathbf{(x + y)(2 - t)}$ ◄

1.7 $tx - ty - 2x + 2y$
$= (tx - ty) - (2x - 2y)$
$= t(x - y) - 2(x - y)$
$= \mathbf{(x - y)(t - 2)}$ ◄

2. Difference between Squares



Check each answer by multiplying back (to the beginning)

2.1 $4x^2 - y^2$
$= \mathbf{(2x + y)(2x - y)}$ ◄

2.2 $4x^2 - 4y^2$
$= 4(x^2 - y^2)$
$= \mathbf{4(x + y)(x - y)}$ ◄

Compare Question 2.1 and Question 2.2!
*In Question 2.1: 4 is **not** a common factor.*
*In Question 2.2: 4 **is** a common factor.*

2.3 $81 - 100a^2$
$= \mathbf{(9 + 10a)(9 - 10a)}$ ◄

2.4 $9p^2 - 36q^2$
$= 9(p^2 - 4q^2)$
$= \mathbf{9(p + 2q)(p - 2q)}$ ◄

2.5 $7x^2 - 28$
$= 7(x^2 - 4)$
$= \mathbf{7(x + 2)(x - 2)}$ ◄

3. Trinomials

3 TERMS

Observe these PRODUCTS:

▹ **Perfect Squares ➡ Perfect Square TRINOMIALS**

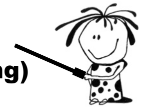
$(x+3)^2 = (x+3)(x+3) = x^2 \mathbf{+ 3x + 3x} + 9 = x^2 \mathbf{+ 6x} + 9$
& $(x-3)^2 = (x-3)(x-3) = x^2 \mathbf{- 3x - 3x} + 9 = x^2 \mathbf{- 6x} + 9$
$\therefore\ x^2 + 6x + 9 = \mathbf{(x + 3)^2}$
& $x^2 - 6x + 9 = \mathbf{(x - 3)^2}$

$(a+b)^2 = (a+b)(a+b) = a^2 \mathbf{+ ab + ab} + b^2 = a^2 \mathbf{+ 2ab} + b^2$
& $(a-b)^2 = (a-b)(a-b) = a^2 \mathbf{- ab - ab} + b^2 = a^2 \mathbf{- 2ab} + b^2$
$\therefore\ a^2 + 2ab + b^2 = \mathbf{(a + b)^2}$
& $a^2 - 2ab + b^2 = \mathbf{(a - b)^2}$

▹ **Other products ➡ TRINOMIALS**

$(x + 2)(x + 3) = \mathbf{x^2 + 2x + 3x + 6} = \mathbf{x^2 + 5x + 6}$
$(x - 2)(x - 3) = \mathbf{x^2 - 2x - 3x + 6} = \mathbf{x^2 - 5x + 6}$
$(x + 2)(x - 3) = \mathbf{x^2 + 2x - 3x - 6} = \mathbf{x^2 - x - 6}$
$(x - 2)(x + 3) = \mathbf{x^2 - 2x + 3x - 6} = \mathbf{x^2 + x - 6}$

Observe the results above to understand factorising trinomials



Check each answer by multiplying back (to the beginning)

3.1 $a^2 + 8a + 16 = (a + 4)(a + 4) = \mathbf{(a + 4)^2}$ ◄

3.2 $p^2 - 10p + 25 = (p - 5)(p - 5) = \mathbf{(p - 5)^2}$ ◄

3.3 $x^2 + 5x + 6 = \mathbf{(x + 3)(x + 2)}$ ◄ . . . $\begin{array}{cc} 1 & +3 \\ 1 & +2 \end{array}$ $\begin{array}{r} +3 \\ +2 \\ \hline +5 \end{array}$

3.4 $x^2 - 5x + 6 = \mathbf{(x - 3)(x - 2)}$ ◄ . . . $\begin{array}{cc} 1 & -3 \\ 1 & -2 \end{array}$ $\begin{array}{r} -3 \\ -2 \\ \hline -5 \end{array}$

3.5 $x^2 + x - 6 = \mathbf{(x + 3)(x - 2)}$ ◄ . . . $\begin{array}{cc} 1 & +3 \\ 1 & -2 \end{array}$ $\begin{array}{r} +3 \\ -2 \\ \hline +1 \end{array}$

3.6 $x^2 - x - 6 = \mathbf{(x - 3)(x + 2)}$ ◄ . . . $\begin{array}{cc} 1 & -3 \\ 1 & +2 \end{array}$ $\begin{array}{r} -3 \\ +2 \\ \hline -1 \end{array}$

3.7 $x^2 - 11x + 18 = \mathbf{(x - 9)(x - 2)}$ ◄ . . . $\begin{array}{cc} 1 & -9 \\ 1 & -2 \end{array}$ $\begin{array}{r} -9 \\ -2 \\ \hline -11 \end{array}$

3.8 $x^2 + 11x + 18 = \mathbf{(x + 9)(x + 2)}$ ◄ . . . $\begin{array}{cc} 1 & +9 \\ 1 & +2 \end{array}$ $\begin{array}{r} +9 \\ +2 \\ \hline +11 \end{array}$

3.9 $x^2 - 7x - 18 = \mathbf{(x - 9)(x + 2)}$ ◄ . . . $\begin{array}{cc} 1 & -9 \\ 1 & +2 \end{array}$ $\begin{array}{r} -9 \\ +2 \\ \hline -7 \end{array}$

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3.10 $x^2 + 7x - 18 = \mathbf{(x+9)(x-2)}$ ➤ . . . $\begin{matrix} 1 & +9 \\ 1 & -2 \end{matrix}$ $\quad \begin{array}{r} +9 \\ -2 \\ \hline +7 \end{array}$

3.11 $x^2 + 9x + 18 = \mathbf{(x+6)(x+3)}$ ➤ . . . $\begin{matrix} 1 & +6 \\ 1 & +3 \end{matrix}$ $\quad \begin{array}{r} +6 \\ +3 \\ \hline +9 \end{array}$

3.12 $x^2 - 9x + 18 = \mathbf{(x-6)(x-3)}$ ➤ . . . $\begin{matrix} 1 & -6 \\ 1 & -3 \end{matrix}$ $\quad \begin{array}{r} -6 \\ -3 \\ \hline -9 \end{array}$

3.13 $x^2 + 3x - 18 = \mathbf{(x+6)(x-3)}$ ➤ . . . $\begin{matrix} 1 & +6 \\ 1 & -3 \end{matrix}$ $\quad \begin{array}{r} +6 \\ -3 \\ \hline +3 \end{array}$

3.14 $x^2 - 3x - 18 = \mathbf{(x-6)(x+3)}$ ➤ . . . $\begin{matrix} 1 & -6 \\ 1 & +3 \end{matrix}$ $\quad \begin{array}{r} -6 \\ +3 \\ \hline -3 \end{array}$

FACTORISATION

There are **3 TYPES** of factorization :

❶ **Common Factor (CF):** Always try this first !

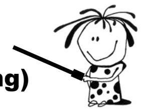
❷ **Difference between Squares (DbS):** 2 terms

❸ **Trinomials:** 3 terms

RECOGNISE THESE !

Mixed Factorisation

Check each answer by multiplying back (to the beginning)



4.1 $3a^3 - 9a^2 - 6a$

$= \mathbf{3a(a^2 - 3a - 2)}$ ➤ . . . *note : this does not factorise further*

4.2 $2a^2 - 18a + 36$

$= 2(a^2 - 9a + 18)$

$= \mathbf{2(a-6)(a-3)}$ ➤ $\quad \begin{matrix} 1 & -6 \\ 1 & -3 \end{matrix}$ $\quad \begin{array}{r} -6 \\ -3 \\ \hline -9 \end{array}$

4.3 $4(a+b) - x^2(a+b)$

$= (a+b)(4 - x^2)$ . . . *always check to see if you can factorise further*

$= \mathbf{(a+b)(2+x)(2-x)}$ ➤

4.4 $6x^3(a-b) + x(b-a)$

$= 6x^3(a-b) - x(a-b)$ . . . *switchround*

$= (a-b)(6x^3 - x)$ . . . *the 'new' factor can factorise further*

$= (a-b) \,.\, x(6x^2 - 1)$

$= \mathbf{x(a-b)(6x^2 - 1)}$ ➤

4.5 $6a^3 - 12a^2 + 18a$

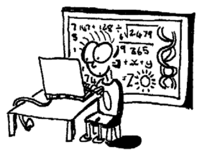
$= \mathbf{6a(a^2 - 2a + 3)}$ ➤ . . . *note : this does not factorise further*

Use factorisation to simplify the following fractions

5.1 $\dfrac{x^2 - 1}{3x + 3}$ . . . **DbS** . . . **CF**

$= \dfrac{\cancel{(x+1)}(x-1)}{3\cancel{(x+1)}}$

$= \mathbf{\dfrac{x-1}{3}}$ ➤



5.2 $\dfrac{x^2 - 4x}{x^2 - 2x - 8}$ . . . **CF** . . . **Trinomial** . . . $\begin{matrix} 1 & -4 \\ 1 & +2 \end{matrix}$ $\quad \begin{array}{r} -4 \\ +2 \\ \hline -2 \end{array}$

$= \dfrac{x\cancel{(x-4)}}{\cancel{(x-4)}(x+2)}$

$= \mathbf{\dfrac{x}{x+2}}$ ➤

5.3 $\dfrac{3a - 6b}{4b - 2a}$

$= \dfrac{3(a - 2b)}{2(2b - a)}$ . . . **Common Factors**

$= \dfrac{3\cancel{(a-2b)}}{-2\cancel{(a-2b)}}$ . . . *2b − a = −(a − 2b)*

$= -\mathbf{\dfrac{3}{2}}$ ➤

5.4 $\dfrac{2x^2 - 8}{3x - 12} \times \dfrac{x^2 - 4x}{x - 2}$

$= \dfrac{2(x^2 - 4)}{3\cancel{(x-4)}} \times \dfrac{x\cancel{(x-4)}}{x-2}$

$= \dfrac{2(x+2)\cancel{(x-2)}}{3\cancel{(x-2)}}$

$= \mathbf{\dfrac{2(x+2)}{3}}$ ➤

Don't be frightened by the look of these fractions !

Just focus on factorising where possible ; then cancel the factors where possible.

5.5 $\dfrac{x^2 + 2x}{x^3 - 2x} \div \dfrac{x^2 - 4}{x - 2}$

$= \dfrac{x^2 + 2x}{x^3 - 2x} \ \mathbf{x}\ \dfrac{x-2}{x^2 - 4}$ . . . *note the 'flipped' fraction !*

$= \dfrac{\cancel{x}\cancel{(x+2)}}{\cancel{x}(x^2 - 2)} \times \dfrac{\cancel{(x-2)}}{\cancel{(x+2)}\cancel{(x-2)}}$

$= \mathbf{\dfrac{1}{x^2 - 2}}$ ➤

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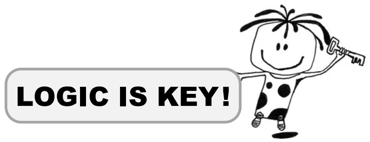
ALGEBRAIC EQUATIONS
(Linear and Quadratic)



1.1 **B** ◄ . . . *If 3 is a root, then x = 3 will make the equation true*

$$\text{i.e.}\quad 3^2 + 3 + t = 0$$
$$\therefore\ 9 + 3 + t = 0$$
$$\therefore\ t = -12$$

1.2 **D** ◄ . . . $2p + 12 = 58$
$\therefore\ 2p = 46$
$\therefore\ p = 23$

OR: $? + 12 = 58$
Answer: 46
So, $2p = 46$
$\therefore\ 2 \times ? = 46$
$\therefore\ p = 23$

1.3 **B** ◄ . . . ***If*** $(x - 1)(x + 2) = 0$,
then $x - 1 = 0$ ***or*** $x + 2 = 0$
$\therefore\ x = 1$ $\therefore\ x = -2$

1.4 **D** ◄ . . . $\dfrac{3x}{2} = -6$
$\therefore\ 3x = -12$
$x = -4$

OR: $\dfrac{?}{2} = -6$
Answer: -12
$\therefore\ 3x = -12$
So, $3 \times ? = -12$
$\therefore\ x = -4$

1.5 **B** ◄ . . . *The product of a number, x, and 6 equals* $x \times 6 = 6x$

2.1 $x + 5 = 2$ . . . *'5 more than a number is 2'*
$\therefore\ \boldsymbol{x = -3}$ ◄

2.2 $x - 3 = -4$ . . . *'3 less than a number is −4'*
$\therefore\ \boldsymbol{x = -1}$ ◄

2.3 $2x = 12$ . . . *'double a number is 12'*
$\therefore\ \boldsymbol{x = 6}$ ◄

2.4 $\dfrac{x}{5} = 6$ . . . *'a fifth of a number is 6'*
$\therefore\ \boldsymbol{x = 30}$ ◄

Check your answer by substituting in the given equation to see if it is 'true' for the value of x.

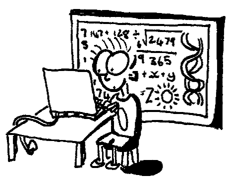
3.1 $3x - 1 = 5$
$\therefore\ 3x = 6$
$\therefore\ \boldsymbol{x = 2}$ ◄

[**Check: LHS** $= 3x - 1 = 3(2) - 1 = 5 =$ **RHS** ✓]

3.2 $2(x + 1) = 10$
$\therefore\ 2x + 2 = 10$
$\therefore\ 2x = 8$
$\therefore\ \boldsymbol{x = 4}$ ◄

. . . OR: $x + 1 = 5$
$\therefore\ x = 4$ ◄

[**Check: LHS** $= 2(4 + 1) = 2 \times 5 = 10 =$ **RHS** ✓]



3.3 $8x + 3 = 3x - 22$
$\therefore\ 8x - 3x = -22 - 3$ . . . *Subtract 3x & Subtract 3*
$\therefore\ 5x = -25$
$\therefore\ \boldsymbol{x = -5}$ ◄ . . . $\left(\div 5,\ \ OR:\ 5 \times \textbf{?} = -25\right)$

Check: **LHS** $= 8(-5) + 3 = -40 + 3 = -37$
& **RHS** $= 3(-5) - 22 = -15 - 22 = -37$
$\therefore$ **LHS** = **RHS** ✓ $\therefore$ The answer is correct.
i.e. The equation is 'true' for $x = -5$.
We say that -5 is the **root** (or solution) of the equation.

3.4 $3(x + 6) = 12$
$\therefore\ 3x + 18 = 12$
$\therefore\ 3x = -6$
$\therefore\ \boldsymbol{x = -2}$ ◄

. . . OR: $x + 6 = 4$
$\therefore\ x = -2$ ◄

Check your answer by substituting in the given equation to see if it is 'true' for the value of x.

3.5 $2x - 5 = 5x + 16$ . . . *Add 5 & Subtract 5x*
$\therefore\ 2x - 5x = 16 + 5$
$\therefore\ -3x = 21$
$\therefore\ \boldsymbol{x = -7}$ ◄ . . . *Divide by −3*

Check your answer by substituting in the given equation to see if it is 'true' for the value of x.

3.6 $x^3 + x^3 = 2$
$\therefore\ 2x^3 = 2$ . . . *LIKE TERMS*
$\therefore\ x^3 = 1$ . . . *divide by 2*
$\therefore\ \boldsymbol{x = 1}$ ◄ . . . *take cube root*

Check your answer by substituting in the given equation to see if it is 'true' for the value of x.

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Equations including fractions

> ***NOTE:*** *In* ***EXPRESSIONS****, the previous section, we did* ***not*** *multiply. We kept the denominator!*
>
> *Now, in* ***EQUATIONS****, we do multiply, applying logic !!! – 'what we do to the left hand side* ***(LHS)*** *of an equation, we also do to the right hand side* ***(RHS)****'.*

4.1 $\frac{x-2}{4} + \frac{2x+1}{3} = \frac{5}{3}$

NB: Brackets!

$$\begin{aligned}
\textbf{x12)} \quad \therefore 3(x-2) + 4(2x+1) &= 4(5) \\
\therefore 3x - 6 + 8x + 4 &= 20 \\
\therefore 11x - 2 &= 20 \\
\therefore 11x &= 22 \\
\therefore \boldsymbol{x} &= \mathbf{2}
\end{aligned}$$

Check your answer!

The LCM of the denominators is 12.
The logic: 12 × the ***LHS*** *= 12 × the* ***RHS***

4.2 $\frac{x+2}{3} - \frac{x-3}{4} = 0$

NB: Brackets!

$$\begin{aligned}
\textbf{x12)} \quad \therefore 4(x+2) - 3(x-3) &= 0 \\
\therefore 4x + 8 - 3x + 9 &= 0 \\
\therefore x + 17 &= 0 \\
\therefore \boldsymbol{x} &= \mathbf{-17}
\end{aligned}$$

Check your answer!

4.3 $\frac{2x-3}{2} - \frac{x+1}{3} = \frac{3x-1}{2}$

NB: Brackets!

$$\begin{aligned}
\textbf{x6)} \quad 3(2x-3) - 2(x+1) &= 3(3x-1) \\
\therefore 6x - 9 - 2x - 2 &= 9x - 3 \\
\therefore 4x - 11 &= 9x - 3 \\
\therefore 4x - 9x &= -3 + 11 \\
\therefore -5x &= 8 \\
\therefore \boldsymbol{x} &= \mathbf{-\frac{8}{5}}
\end{aligned}$$

6 times the ***LHS*** *= 6 times the* ***RHS***

Check your answer!

4.4 $\frac{x}{1} - \frac{x-1}{2} = \frac{3}{1}$

NB: Brackets!

$$\begin{aligned}
\textbf{x2)} \quad \therefore 2x - (x-1) &= 2(3) \\
\therefore 2x - x + 1 &= 6 \\
\therefore x + 1 &= 6 \\
\therefore \boldsymbol{x} &= \mathbf{5}
\end{aligned}$$

2 times the ***LHS*** *= 2 times the* ***RHS***

Check your answer!

4.5 $\frac{x+1}{3} - \frac{x-1}{6} = \frac{1}{1}$

NB: Brackets!

$$\begin{aligned}
\textbf{x6)} \quad \therefore 2(x+1) - (x-1) &= 6(1) \\
\therefore 2x + 2 - x + 1 &= 6 \\
\therefore x + 3 &= 6 \\
\therefore \boldsymbol{x} &= \mathbf{3}
\end{aligned}$$

6 times the ***LHS*** *= 6 times the* ***RHS***

Check your answer!

> **Compare the position of the = signs**
>
> ***In Algebraic Expressions*** *(in the previous section):*
> *The* **= *signs*** *are down the* ***left***
>
> ***In Algebraic Equations*** *(Q4.1 to 4.5 above):*
> *The* **= *signs*** *are in the* ***middle***
> *and the* $\therefore$ ***signs*** *are on the* ***left***

Quadratic Equations

> **The LOGIC:**
> **If the product of 2 numbers = 0, then either one or the other number must = 0.**

5.1 $(x-3)(x+4) = 0$ ... *the product of $x-3$ and $x-4$ equals 0, so:*

Either $x - 3 = 0$ **or** $x + 4 = 0$

$\therefore \boldsymbol{x} = \mathbf{3}$ $\quad\quad$ $\therefore \boldsymbol{x} = \mathbf{-4}$

Check: If $x = 3$:
LHS $= (3-3)(3+4) = 0(7) = 0 =$ **RHS** ✓

If $x = -4$:
LHS $= (-4-3)(-4+4) = (-7) \times 0 = 0 =$ **RHS** ✓

$\therefore$ Both answers are correct

5.2 $x^2 - 5x - 6 = 0$ ... *Factorise the* ***trinomial*** *so that you have a product*

$\therefore (x-6)(x+1) = 0$

$\therefore$ **Either** $x - 6 = 0$ **or** $x + 1 = 0$

$\therefore \boldsymbol{x} = \mathbf{6}$ $\quad\quad$ $\boldsymbol{x} = \mathbf{-1}$

$$\begin{array}{ccc} 1 & \times & -6 \\ 1 & + & 1 \end{array} \qquad \begin{array}{r} -6 \\ +1 \\ \hline -5 \end{array}$$

Check your answers!

5.3 $x^2 - 1 = 0$ ... *Factorise!* ***(Difference between squares)***

$\therefore (x+1)(x-1) = 0$

$\therefore$ **Either** $x + 1 = 0$ **or** $x - 1 = 0$

$\therefore \boldsymbol{x} = \mathbf{-1}$ $\quad\quad$ $\boldsymbol{x} = \mathbf{1}$

OR: $x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1$

Check your answers!

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5.4 $\quad x^2 - 2x = 0$ $\quad$ . . . *Factorise !* ***(Common Factor)***

$\therefore\ x(x-2) = 0$

$\therefore$ **Either** $\boldsymbol{x = 0}$ ➤ $\quad$ **or** $\quad x - 2 = 0$

$\therefore\ \boldsymbol{x = 2}$ ➤

Check your answers!

In Questions 6.1 and 6.2 :

$(x-2)^2$
$= (x-2)(x-2)$
$= x^2 - \mathbf{2x} - \mathbf{2x} + 4$
$= x^2 - \mathbf{4x} + 4$

In Question 6.3 :

$(x+3)^2$
$= (x+3)(x+3)$
$= x^2 + \mathbf{3x} + \mathbf{3x} + 9$
$= x^2 + \mathbf{6x} + 9$

Now, the equations . . .

6.1 $\quad 2(x-2)^2 = (2x-1)(x-3)$

$\therefore\ 2(x-2)(x-2) = 2x^2 - 6x - x + 3$

$\therefore\ 2(x^2 - 4x + 4) = 2x^2 - 7x + 3$

$\therefore\ 2x^2 - 8x + 8 = 2x^2 - 7x + 3$

$\therefore\ -8x + 7x = 3 - 8$ $\quad$ . . . *the two $2x^2$ terms cancel each other and the equation becomes* ***linear*** *(no longer* ***quadratic****).*

$\therefore\ -x = -5$

$\therefore\ \boldsymbol{x = 5}$ ➤

Check your answer!

6.2 $\quad (x-2)^2 + 3x - 2 = (x+3)^2$

$\therefore\ (x-2)(x-2) + 3x - 2 = (x+3)(x+3)$

$\therefore\ x^2 - 4x + 4 + 3x - 2 = x^2 + 6x + 9$

$\therefore\ -x + 2 = 6x + 9$ $\quad$ . . . *the two $2x^2$ terms cancel each other and the equation becomes* ***linear****.*

$\therefore\ -x - 6x = 9 - 2$

$\therefore\ -7x = 7$

$\therefore\ \boldsymbol{x = -1}$ ➤

Check your answer!

6.3 $\quad (x-3)^2 = 16$

$(x-3)(x-3) = 16$

$\therefore\ x^2 - 6x + 9 - 16 = 0$

$\therefore\ x^2 - 6x - 7 = \mathbf{0}$ ⬅ *Remember the logic ? The product must* ***= 0*** *!*

$\therefore\ (x-7)(x+1) = 0$ ⬅ *The* ***trinomial*** *is factorised*

The logic :

$(x-7)$ times $(x+1)$ equals 0, so . . . .

Either $\ x - 7$ equals 0 $\qquad$ **or** $\qquad x + 1$ equals 0

i.e. **Either** $\ x - 7 = 0$ $\qquad$ **or** $\qquad x + 1 = 0$

$\therefore\ \boldsymbol{x = 7}$ ➤ $\qquad$ **or** $\qquad \boldsymbol{x = -1}$ ➤

Check your answers!

Other . . .

Note :
Solving equations requires ***reversing operations***

+ ⟷ -	**x ⟷ ÷**	**powers ⟷ roots**
$x \mathbf{+3} = 8$ $\therefore\ x + 3 \mathbf{-3} = 8 \mathbf{-3}$ $\therefore\ x = 5$ ➤	$\mathbf{3}x = 12$ $\therefore\ \dfrac{3x}{\mathbf{3}} = \dfrac{12}{\mathbf{3}}$ $\therefore\ x = 4$ ➤	$x^{\mathbf{3}} = 8$ Take $\sqrt[3]{\ }$ on both sides $\therefore\ x = 2$ ➤ ***Note :*** $x^{\mathbf{2}} = 9$ $\therefore\ x = \pm 3$ *If the power is even, there are 2 roots !*
$x \mathbf{-3} = 8$ $\therefore\ x - 3 \mathbf{+3} = 8 \mathbf{+3}$ $\therefore\ x = 11$ ➤	$\dfrac{x}{\mathbf{3}} = 12$ $\therefore\ \dfrac{x}{3}\ \mathbf{x\,3} = 12\ \mathbf{x\,3}$ $\therefore\ x = 36$ ➤	$\sqrt{x} = 5$ **Square** both sides $(\sqrt{x})^2 = 5^2$ $\therefore\ x = 25$ ➤

ALWAYS CHECK YOUR ANSWER !

So, solve for x : (a) $\quad \sqrt{x} = 2$ $\qquad$ (b) $\quad \sqrt{\sqrt{x}} = 2$

(a) *Square both sides*

$\therefore\ x = 4$ ➤

(b) *Square both sides*

$\therefore\ \sqrt{x} = 4$

Square both sides again

$\therefore\ x = 16$ ➤

Now see Q7.1

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7.1

$$\sqrt{\sqrt{\sqrt{x}}} = 2$$

Square both sides $\quad \therefore \left(\sqrt{\sqrt{\sqrt{x}}}\right)^2 = (2)^2$

$$\therefore \sqrt{\sqrt{x}} = 4$$

Square both sides again $\quad \therefore \left(\sqrt{\sqrt{x}}\right)^2 = (4)^2$

$$\therefore \sqrt{x} = 16$$

Square both sides again ! $\quad \therefore \left(\sqrt{x}\right)^2 = (16)^2$

$$\therefore \boldsymbol{x} = \mathbf{256}$$

Check your answer!

7.2

$$\sqrt{\frac{1}{\sqrt{x}}} = 2$$

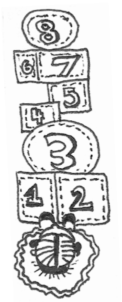
$$\therefore \left(\sqrt{\frac{1}{\sqrt{x}}}\right)^2 = (2)^2$$

$$\therefore \frac{1}{\sqrt{x}} = 4$$

$$\therefore \left(\frac{1}{\sqrt{x}}\right)^2 = (4)^2$$

$$\therefore \frac{1}{x} = \frac{16}{1}$$

$$\therefore \boldsymbol{x} = \mathbf{\frac{1}{16}}$$



Check your answer!

NOTES

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GRAPHS

1.1 **B** ➤ . . . *$f(x) = 2x + 4$:*

- *positive gradient of $\frac{2}{1}$* . . . $\mathbf{2}x$,
- *y–int of 4* . . . *$y = 4$ when $x = 0$*

If a point lies on a line, then the **equation** of the graph will be **true** for its coordinates. *(See Question 1.2)*

1.2 **D** ➤ . . . *The equation is $y = x$, so x and y will have to be equal (i.e. the coordinates must have the same value)*

1.3 **B** ➤ . . . $\frac{d}{6} = \frac{2}{3}$ $\therefore$ *$d = 4$* . . . *equivalent fractions*

Very important to know:

On the **Y-axis**, the $\boldsymbol{x}$-coordinate is (always) **0** *(See Question 1.4)*

On the **X-axis**, the **y**-coordinate is (always) **0** *(See Question 1.5)*

1.4 **C** ➤ . . . *Substitute $x = 0$; then*

y–intercept: $4(0) + 2y = 12$

$\therefore\ 2y = 12$

$\therefore\ y = 6$

So, the point on the y–axis is (0; 6)

1.5 **B** ➤ . . . *Substitute $y = 0$; then*

x–intercept: $3(0) + 2x + 1 = 0$

$\therefore\ 2x = -1$

$\therefore\ x = -\frac{1}{2}$

$\therefore\ \left(-\frac{1}{2}; 0\right)$ . . . *the coordinates of the x–intercept*

2. P is the intersection of the lines $y = x$ and $y = 3$ and so at point P, both these equations must 'be true'.

So, y must equal x and y must equal 3. $\therefore\ y = x = 3$

$\therefore$ **P(3; 3)** ➤

3.1 $y = 3x - 5$

x	-2	-1	0	1
y	-11	-8	-5	-2

$y = 3(-2) - 5 = -11$

$y = 3(-1) - 5 = -8$

$y = 3(0) - 5 = -5$

$y = 3(1) - 5 = -2$

We substitute the values of x into the equation to find y.

3.2 $y = -\frac{2}{3}x - 1$

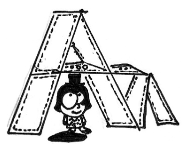
x	-3	-1	0	1
y	1	$-\frac{1}{3}$	-1	$-\frac{5}{3}$

$y = -\frac{2}{3}\left(-\frac{3}{1}\right) - 1 = 2 - 1 = 1$

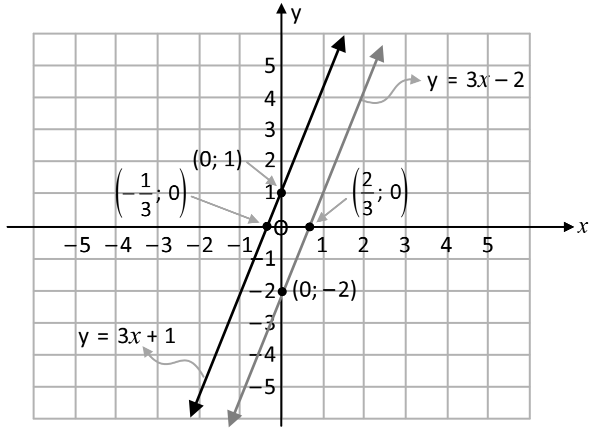
$y = -\frac{2}{3}(-1) - 1 = \frac{2}{3} - 1 = -\frac{1}{3}$

$y = -\frac{2}{3}(0) - 1 = -1$

$y = -\frac{2}{3}(1) - 1 = -\frac{2}{3} - 1 = -\frac{5}{3}$



4.1



To find the points where the graphs cut the axes:

$\mathbf{y = 3x - 2}$: | **$\mathbf{y = 3x + 1}$:**

For the **Y-intercept**, substitute $\boldsymbol{x = 0}$

$y = 3x - 2$:

$\therefore\ y = 3(0) - 2$

$= -2$

$\therefore$ The graph cuts the **y–axis** at -2.

The **point** is (**0**; 2)

$y = 3x + 1$:

$\therefore\ y = 3(0) + 1$

$= 1$

$\therefore$ The graph cuts the **y–axis** at 1.

The **point** is (**0**; 1)

For the **X-intercept**, substitute **y = 0**

$y = 3x - 2$:

$\therefore\ 0 = 3x - 2$

$\therefore\ 3x = 2$

$\therefore\ x = \frac{2}{3}$

$\therefore$ The graph cuts the $\boldsymbol{x}$**–axis** at $\frac{2}{3}$.

The **point** is $\left(\frac{2}{3}; \mathbf{0}\right)$

$y = 3x + 1$:

$\therefore\ 0 = 3x + 1$

$\therefore\ 3x = -1$

$\therefore\ x = -\frac{1}{3}$

$\therefore$ The graph cuts the $\boldsymbol{x}$**–axis** at $-\frac{1}{3}$.

The **point** is $\left(-\frac{1}{3}; \mathbf{0}\right)$

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4.2 They are parallel ➤ *. . . they have equal gradients*

5.1 **AD:** The gradient $= -\frac{4}{2} = -2$ *. . .* $\therefore\ m = -2$

& the y–intercept is 4 *. . .* $\therefore\ c = 4$

$\therefore$ The equation is $y = -2x + 4$ ➤ *. . .* $m = -2\ \&\ c = 4$ *in* $y = mx + c$

BC: The gradient $= -\frac{4}{2} = -2$ *. . .* $\therefore\ m = -2$

& the y–intercept is -4 *. . .* $\therefore\ c = -4$

$\therefore$ The equation is $y = -2x - 4$ ➤ *. . .* $m = -2\ \&\ c = -4$ *in* $y = mx + c$

The standard form of the equation of a straight line is $\mathbf{y = mx + c}$, where $\mathbf{m}$ = the gradient and $\mathbf{c}$ = the y–intercept.

5.2 They are parallel.
They both have gradients of -2.

Both gradients are negative and are measured as $\dfrac{\textit{number of units down}}{\textit{number of units across}}$

i.e. $\dfrac{\textit{vertical change}}{\textit{horizontal change}}$

6.1 The gradient $= -\frac{5}{1} = \mathbf{-5}$ ➤

By inspection
- negative gradient
- $\dfrac{\text{rise}}{\text{run}}$ or $\dfrac{\text{vertical change}}{\text{horizontal change}}$

So, — and $\dfrac{\textbf{5 units down}}{\textbf{1 unit across}}$

The use of a formula for the gradient is not ideal for grade 9 learners.

6.2 $\mathbf{y = -5x + 5}$ ➤ *. . . gradient,* $m = -5$ & *y–intercept,* $c = 5$

6.3 The gradient of any other straight line drawn parallel to this line is -5. ➤ *. . . parallel lines have the same gradient*

7.1

	A	B	C
x–coordinate	**0**	2	4
y–coordinate	-2	**0**	2

Note: ***x* = 0** *on the* ***y–axis*** *(at A)*
& ***y* = 0** *on the* ***x–axis*** *(at B)*

7.2 $\mathbf{y = x - 2}$ ➤ . . . By inspection: The y–coordinates are all 2 less than the x–coordinates.

or: Gradient $= +\frac{2}{2} = 1$
& y–intercept, $c = -2$

8.1 The equation of CD: $\mathbf{x = 2}$ ➤

. . . because every point on (vertical) line CD has an x–coordinate equal to 2

$\therefore$ *x = 2 is the* ***equation*** *of CD*

The equation of a line is a 'rule' which is true for all points on the line.

8.2 The equation of AB: $\mathbf{y = 2x}$ ➤

Method 1:
Observe various points on the graph:
e.g. $(-2; -4)$; $(-1; -2)$; $(1; 2)$
and notice that y always equals twice x

Method 2:
m, the gradient $= +\frac{2}{1} = 2$
& c, the y–intercept, is 0

8.3 **E(2; −2)** ➤ *. . .* $x = 2$ *and* $y = -2$ *at point E*

8.4 CE = **6 units** ➤ *. . . CE = CD + DE = 4 + 2 = 6 units*

OR $CE = Y_C - Y_E$ *. . . the difference of the y–coordinates of C and*
$= 4 - (-2)$
$= 6$

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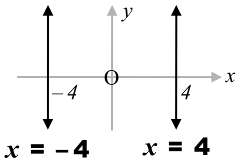
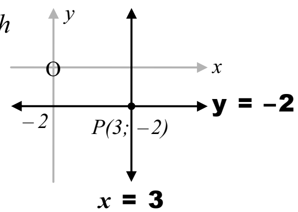
9.1 The lines $x = 4$ and $x = -4$ are **parallel** to one another. ◀

. . . The lines $\boldsymbol{x = 4}$ *and* $\boldsymbol{x = -4}$*:*
are both parallel to the y–axis



9.2 The equation of the horizontal line through the point P(3; –2) is **y = –2**. ◀

. . . The horizontal line through P(3; –2) is $\boldsymbol{y = -2}$*;*
The vertical line through P(3; –2) is $\boldsymbol{x = 3}$*;*



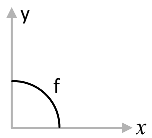
9.3 The gradient of the line defined by $y - 4x + 5 = 0$ is equal to **4**. ◀

$. . . y - 4x + 5 = 0$

$\therefore\ y = \mathbf{4}x - 5 \qquad . . . y = \mathbf{m}x + c$

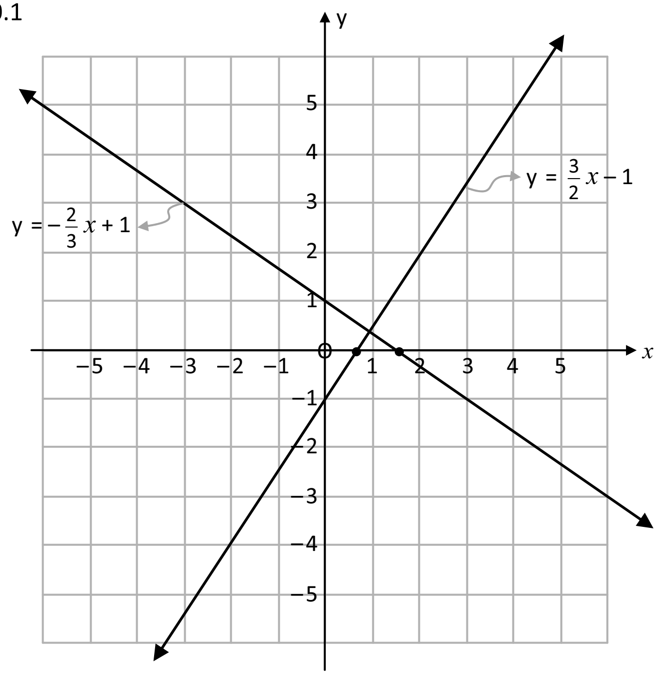
$\therefore$ *The gradient, which is the coefficient of x, is* **4**

9.4 This graph of f below represents a **non–linear** function. ◀



. . . A linear function is
a straight line,
not a curve.

10.1



To find the points where the graphs cut the axes:

$\boldsymbol{y = -\frac{2}{3}x + 1:}$	$\boldsymbol{y = \frac{3}{2}x - 1:}$

For the **Y-intercept**, substitute $\boldsymbol{x = 0}$

$y = -\frac{2}{3}x + 1$:

$\therefore\ y = -\frac{2}{3}(0) + 1$

$= 1$

$\therefore$ The graph cuts the **y–axis** at 1.

The **point** is (**0**; 1)

$y = \frac{3}{2}x - 1$:

$\therefore\ y = \frac{3}{2}(0) - 1$

$= -1$

$\therefore$ The graph cuts the **y–axis** at –1.

The **point** is (**0**; –1)

For the **X-intercept**, substitute **y = 0**

$y = -\frac{2}{3}x + 1$:

$\therefore\ 0 = -\frac{2}{3}x + 1$

$\therefore\ \frac{2}{3}x = 1$

$\therefore\ 2x = 3 \qquad . . . \times 3$

$\therefore\ x = \frac{3}{2} \qquad . . . \div 2$

$\therefore$ The graph cuts the $\boldsymbol{x}$**–axis** at $\frac{3}{2}$.

The **point** is $\left(\frac{3}{2};\ \mathbf{0}\right)$

$y = \frac{3}{2}x - 1$:

$\therefore\ 0 = \frac{3}{2}x - 1$

$\therefore\ \frac{3}{2}x = 1$

$\therefore\ 3x = 2 \qquad . . . \times 2$

$\therefore\ x = \frac{2}{3} \qquad . . . \div 3$

$\therefore$ The graph cuts the $\boldsymbol{x}$**–axis** at $\frac{2}{3}$.

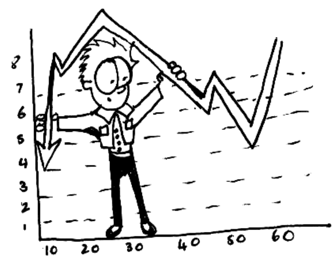
The **point** is $\left(\frac{2}{3};\ \mathbf{0}\right)$

10.2 They are perpendicular.

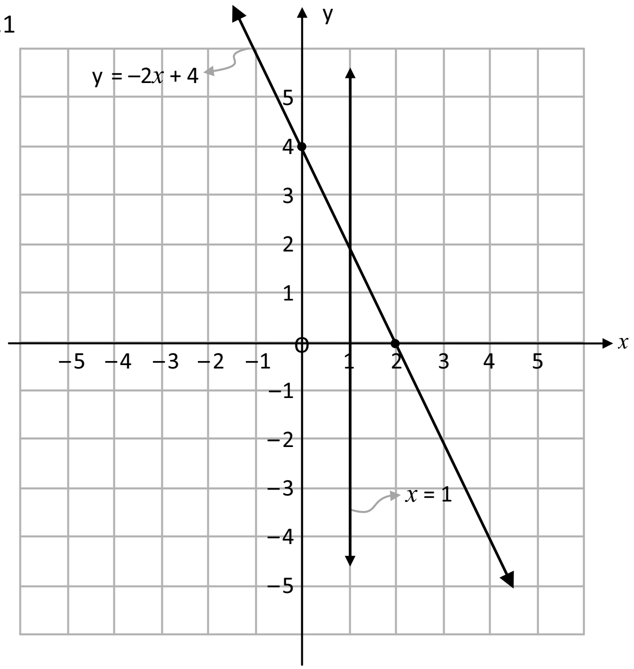
Out of interest:

Compare the gradients, $-\frac{2}{3}$ and $\frac{3}{2}$.

They are negative inverses of one another.



11.1



To find the points where the graphs cut the axes:

y = −2x + 4:

y–intercept (substitute **x = 0**): $y = -2(\mathbf{0}) + 4$
$= 4$

x–intercept (substitute **y = 0**): $\mathbf{0} = -2x + 4$
$\therefore\ 2x = 4$
$\therefore\ x = 2$

x = 1: This graph is a **vertical** line through $x = 1$. Every point on the graph has an x–coordinate equal to 1.

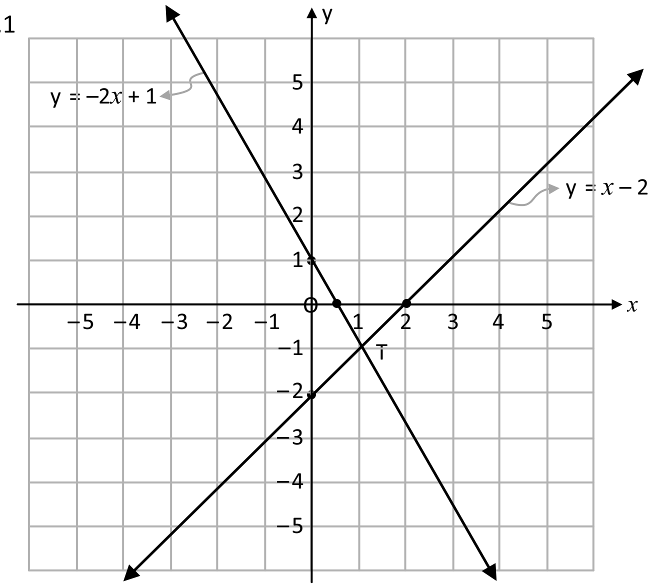
11.2 The point of intersection is (1; 2)

. . . At this point, $x = 1$ and $y = -2x + 4$
(i.e. both equations are true)

$\therefore\ y = -2(1) + 4$
$= 2$

∴ The point is (1; 2)

12.1



To find the points where the graphs cut the axes:

y = −2x + 1:	**y = x − 2:**
For the **Y-intercept**, substitute **x = 0**	
$\therefore\ y = -2(0) + 1$ $= 1$	$\therefore\ y = (0) - 2$ $= -2$
∴ The graph cuts the **y–axis** at 1.	∴ The graph cuts the **y–axis** at −2.
The **point** is (**0**; 1)	The **point** is (**0**; −2)
For the **X-intercept**, substitute **y = 0**	
$\therefore\ 0 = -2x + 1$ $\therefore\ 2x = 1$ $\therefore\ x = \frac{1}{2}$	$\therefore\ 0 = x - 2$ $\therefore\ x = 2$
∴ The graph cuts the **x–axis** at $\frac{1}{2}$.	∴ The graph cuts the **x–axis** at 2.
The **point** is $\left(\frac{1}{2};\ \mathbf{0}\right)$	The **point** is (2; **0**)

12.2 $y = -2x + 1$ . . . ❶ & $y = x - 2$ . . . ❷

❶ = ❷: $-2x + 1 = x - 2$
$\therefore\ -2x - x = -2 - 1$
$\therefore\ -3x = -3$
$\therefore\ x = 1$

Both equations must be true at T, the point of intersection.

Substitute $x = 1$ into ❷: $y = (1) - 2$ [OR into ❶!]
$= -1$

∴ T(1; −1)

NOTES

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